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TO THE THEORY OF GRADIENT METHOD OF DETERMINATION THE MEAN INTEGRAL REFRACTIVE INDEX OF AIR AT LONG DISTANCE MEASUREMENTS ON THE NEAR-EARTH TRACES

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Formulation of the problem.

Despite intensive research into the effects of the earth's atmosphere on the results of distance measurements carried out with the help of electromagnetic waves on surface paths, these effects are still one of the most significant factors limiting the accuracy of such measurements. These effects include, as is known [1-3], the difference between the speed of signal propagation in the atmosphere and the speed of light in a vacuum, as well as the curvature of the trajectory along which the signal propagates due to the effect of refraction.

These effects can be compensated either by introducing the corresponding independently determined corrections into the measurement results, or by instrumental means directly during the measurements. The analysis of promising methods for determining the corrections for the refractional curvature of the trajectory (which usually affects the measurement accuracy much less than the difference between the signal propagation speeds in the medium and the vacuum) was performed in [4].

As for the compensation of the influence of the atmosphere on the speed of signal propagation, to which the main attention will be devoted further, it is carried out using the average integral along the measured path of the refractive index

$$\bar{n} = D^{-1}\int_{0}^{\sigma_D} n(r(\sigma))d\sigma \,. \qquad (1)$$

The integration in the formula (1) is performed along the radial trajectory (from the initial point of the trajectory = 0 to the final = D); the shape of the trajectory is determined by the ray equation of geometric optics [5]; n (r) is the dependence of the refractive index of air n on the coordinates r = r (x, y, z); D is the length of the trajectory corresponding to the length of the measured line.

The determination of the value is possible both on the basis of the model representations of the profile n (r) and integral (1), and by instrumental means directly during the measurements. Among the model representations, the so-called point approximation methods are often used, based on the representation of the integrand function or the integral (1) itself as a sum of terms determined at certain fixed points of the measured trace. For the instrumental approach, the most well-known are geodesic and dispersion methods [1-3].

Given the complexity of the practical implementation, as well as the limitations laid down in the basis of instrumental methods (in particular, the neglect of the refractive spatial spread of the trajectories of signals with different optical carrier wavelengths - for dispersive methods [2], as well as the use of the concept of average integral refraction coefficient for geodetic methods [3]), this article discusses the accuracy capabilities of model methods using approximation principles, namely: methods based on the representation integral (1) finite sum. The most studied among such methods are the methods that use the trapezoid and Euler-Maclaurin formulas for the approximate representation of integral (1), in particular, the gradient method [6–8].

Further, the main results achieved to date in the study of these methods are analyzed, the prospects for their further development are discussed.

The presentation of the main material.

Using the Euler-Maclaurin integration formula, based on the integral representations of the ray equations of geometric optics proposed in [9, 10], relation (1) in the case of the gradient method for determining the average integral air refractive index can be represented as [6-8]:

$$\bar{n} = \bar{n}_T - \frac{D}{12N^2}\Big(g_{G_D}\cdot\sin Z_D - g_{V_D}\cdot\cos Z_D - - g_{G_0}\cdot\sin Z_0 - g_{V_0}\cdot\cos Z_0\Big) + R_{EM} \qquad (2)$$

где

$$\bar{n}_T = \left[\frac{n_0+n_D}{2} + \sum_{i=1}^{N-1} n(\sigma_i)\right]\cdot\frac{1}{N} \quad - \qquad (3)$$

- the ratio that coincides with the working formula for the average integral refractive index of air, determined by the trapezium method;

$\sigma_i = D\cdot\frac{i}{N}$; $i = 1,2,\ldots N-1$: N - number of breaks in the integration interval ($N+1$ – the number of points at which measurements of local values of the refractive index are performed); z_L and z_0 - angles of vertical refraction at the end points of the path; g_{G_L}, g_{V_L}, g_{G_0} and g_{V_0} - the values of the horizontal and vertical projections of the gradient of the refractive index of air at these points; R_{EM} - residual term of the Euler-Maclaurin integration formula.

It is easy to see that the relation (2) of the gradient method refines the relation (3) of the well-known method of point approximation by taking into account the results of measurements of the gradients of the refractive index of air and the refraction angles at the end points of the path (which, in principle, allows reducing the number of intermediate points of the path in which local values of the refractive index of air). It also generalizes the well-known geodesic method of A. L. Ostrovsky [3] due to additional measurements at intermediate points of the route.

Increasing the accuracy of the gradient method, the measurement equation of which is based on relation (2), as compared to the trapezoidal method (formula (3)), can be established by comparing the residual terms in integral representations of (1) with finite sums for these methods [11, 12] . For the gradient method (2), the residual term is

$$R_{EM}=\frac{D^4}{720N^4}\cdot\frac{d^4n}{d\sigma^4}\Big|_{\sigma=\sigma_*}\quad, \tag{4}$$

where σ_* - some point of the trajectory in the integration interval, and for the point approximation method using the trapezoidal formula (3) this will be the ratio

$$R_T=\frac{D^2}{12\cdot N^2}\cdot\frac{d^2n}{d\sigma^2}\Big|_{\sigma=\sigma_{**}}\quad, \tag{5}$$

where σ_{**} - also some point on the integration interval. If a

$$|R_{EM}|\langle\langle|R_T|, \tag{6}$$

This gradient method has a higher potential accuracy than the one based on the trapezoid formula.

To verify the validity of this statement, we substitute (4), (5) into (6) under the assumption that the function $n(\sigma)$ near points σ_*, σ_{**} , in which residual terms are defined (4) (5), changes according to the law

$$n(\sigma)=\begin{cases} n_*\cdot e^{-g_V(\sigma-\sigma_*)}-\textit{вблизи}\quad\sigma_* \\ n_{**}\cdot e^{-g_V(\sigma-\sigma_{**})}-\textit{вблизи}\quad\sigma_{**}\end{cases}$$

Then, considering that, at km -1 (this value is typical for the vertical gradient of the refractive index of air in normal conditions), = 1 ... 100 km (the possible length range for laser measurements on near-earth paths), (when all measurements are carried out only in end points of the trace), we find that the value of the right side of inequality (6) significantly exceeds the value of the left side. Thus, on the basis of estimates of the residual terms, the order of magnitude of which is determined by the order of the derivative of the integrand function, we can conclude that the formula of the gradient method for determining the average integral refractive index of air is characterized by much greater potential accuracy than the formula of the trapezium method. Quantitative estimates of the error of these formulas can be specified on the basis of a numerical or full-scale experiment with real profiles of the refractive index of air.

The disadvantages of the above methods, the working relations of which are reflected in formulas (2), (3), include the requirement of uniform placement along the integration interval of points for determining the local values of the refractive index of air necessary for the approximate calculation of integral (1). This requirement is not always feasible in practical measurement conditions (in particular, when measuring over a strongly inhomogeneous underlying surface). To eliminate this drawback, a variant of the gradient method is considered below with nonuniform splitting of the integration interval.

In [13], it was shown that the working relations of this method can be obtained based on the calculation of integral (1) using the Hermite interpolation polynomial [11, 12] for an approximate representation of the dependence of the refractive index of air on the ray coordinate

$$n_m(\sigma)=\sum_{i=0}^{N}\sum_{j=0}^{\alpha_i-1}\sum_{k=0}^{\alpha_i-j-1}n_i^{(j)}\cdot\frac{1}{k!}\cdot\frac{1}{j!}\cdot\left[\frac{(\sigma-\sigma_i)^{\alpha_i}}{\Omega(\sigma)}\right]^{(k)}\cdot$$

$$\cdot\frac{\Omega(\sigma)}{(\sigma-\sigma_i)^{\alpha_i-j-k}}\quad, \tag{7}$$

The remainder term for the above Hermite interpolation formula (7) is

$$R_E=n(\sigma)-n_m(\sigma)=\frac{n^{m+1}(\sigma_{***})}{(m+1)!}\cdot\Omega(\sigma),$$

where the point belongs to the integration interval.

Relation (7) is simplified in the case of, that is, when derivatives are used for interpolating functions only at the start and end points of the path (as in the gradient method based on formula (2)). In this practically important case, the desired value can be represented by the ratio

$$\bar{n}\cdot D=\sum_{i=1}^{N-1}A_i\cdot n(\sigma_i)+\sum_{i=0,N}\left[A_{i0}\cdot n(\sigma_i)+A_{i1}\cdot n^{(1)}(\sigma_i)\right]+$$

$$+\int_0^D R_E\cdot d\sigma\ ,\quad \text{где} \tag{8}$$

$$A_i=\left[\frac{\sigma-\sigma_i}{\Omega(\sigma)}\right]_{\sigma=\sigma_i}\cdot\int_0^D\frac{\Omega(\sigma)}{\sigma-\sigma_i}\cdot d\sigma\quad \text{при } i=1,\ldots,N-1,$$

$$A_{i0}=\left[\frac{(\sigma-\sigma_i)^2}{\Omega(\sigma)}\right]_{\sigma=\sigma_i}\cdot\int_0^D\frac{\Omega(\sigma)\cdot d\sigma}{(\sigma-\sigma_i)^2}+$$

$$+\left[\frac{(\sigma-\sigma_i)^2}{\Omega(\sigma)}\right]^{(1)}_{\sigma=\sigma_i}\cdot\int_0^D\frac{\Omega(\sigma)\cdot d\sigma}{\sigma-\sigma_i}\qquad \text{при } i=0,N\ ,$$

$$A_{i1} = \left[\frac{(\sigma - \sigma_i)^2}{\Omega(\sigma)} \right]_{\sigma = \sigma_i} \cdot \int_0^D \frac{\Omega(\sigma) \cdot d\sigma}{\sigma - \sigma_i} \quad \text{при} \quad i = 0, N \ .$$

Further simplifications are realized when, when only values for the start and end points of the path are used for the determination. Using formula (8), we then obtain an analytical relation that completely coincides with formula (2) of the gradient method with (taking into account the relationship of the derivative of the ray coordinate of the refractive index with the gradient of the refractive index and the angle of refraction [6,10]). The coincidence of the residual terms of these relations indicates the comparable potential accuracy of both methods for measurements carried out only at the end points of the path.

If, however, and for determining except at the end points, the value at some intermediate point is used, then the integrals in (8) are also represented by analytic, but rather cumbersome functions (which for this reason are not given here). Note that the limiting transition to the Euler-Maclaurin quadrature formula (2) in this case is not satisfied even under the condition that the trace is evenly divided. Indeed, for formula (8) gives a relation that no longer coincides with (2) with

$$\bar{n} = \frac{1}{30} \cdot \left[7(n_0 + n_D) + 16n\left(\sigma = \frac{D}{2}\right) \right] + \frac{D}{60} \cdot \left[n_0^{(1)} - n_D^{(1)} \right]$$

(in which to reduce the entry is not given the residual term). It is essential, however, that this residual term will be proportional to the derivative of the fifth order, and not the fourth, as in the case of the Euler-Maclaurin decomposition with (formula (4)). Moreover, the increase in the number of intermediate points in the case of Hermite interpolation, in contrast to the case of Euler-Maclaurin, increases accordingly the order of the derivative in the remainder term. This means that the use of Hermite polynomials improves the potential accuracy of the gradient method.

A more detailed analysis of the accuracy of the method under discussion can be carried out using model or experimental data on the spatial profiles of the refractive index of air on the measured path.

Conclusions and offers.

An analysis was made of ways to improve the accuracy of model methods for taking into account the average integral refractive index of air when measuring distances carried out using electromagnetic waves on surface paths.

The possibilities and prospects for the further development of the gradient method, which refines the well-known point approximation method - based on additional information about the gradients of the refractive index of air and the refraction angles at the end points of the measured path, and the well-known geodetic method of A. L. Ostrovsky are considered - due to additional measurements of local values refractive index of air at intermediate points of the route.

The possibility of modifying the gradient method to the case of non-uniform splitting of the measured trace by points, in which local values of the refractive index are determined, is shown. It is of interest to study the accuracy of the gradient method using experimental data on the profiles of the refractive index on surface paths**.**

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An analysis of the accuracy of the gradient method for determining the mean integral refraction index of air at range measurements on the surface tracks is executed. It is shown the possibility of modifying this method to the case of uneven partition of the measured trace by points, in which the local values of the refractive index are determined.