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THE INVARIANTS OF PSC AS CRITERIA OF CARTOGRAPHIC CORRECTNESS OF ARCHIVAL TOPOGRAPHIC MAPS

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Formulation of the

In modern conditions, it is required that the urban topographic network corresponds to the current state of the territory [1, 2]. The topographic basis is created (updated) mainly using topographic maps, created 20-30 years ago [3, 4]. In order to create a city GIS had reliable and complete topographic and geodetic information, it is necessary to evaluate the content of archival map material. One of the methods is to create plans for solid (stable, durable) contours.

Presentation of the main material of the problem

The plan of solid contours (PSC) is a contour specialized topographical plan, which is created in digital form, the accuracy of which corresponds to the accuracy of the topographic plan of scale 1: 500.

On the basis of the PSC, there can be obtained quantitative invariant characteristics of the stability or dynamics of the studied areas. Such characteristics are of particular importance for small towns and district centers, which, in most cases, do not have enough money to switch to digital technology.

An analysis of recent researches and publications that refer this

Relevant researches are topical and timely. Except [5] there are no publications on this topic in the literature.

Unresolved parts of the general problem

One of the possible methods for analyzing archival top materials is the creation of a PTC. In the case of matching contours, it is expedient to use the mathematical apparatus of the mean-square prediction of Kolmogorov-Wiener.

Problem statement

To substantiate theoretically invariant estimates of PSC; on a concrete example to obtain values of deformation as a result of raster scanning; to illustrate the method of analysis of archival cartographic materials and the possibility of their use in modern geodetic production.

Presentation of the main material of the problem

1. Invariant definitions. In the analysis of multi-time maps, it is important to evaluate correctly the deformation (distortion) [6, 7]. To do this, we offer a modified version of calculations of indications of Tissot.

The literature discusses in detail the method of determining the parameters of the indications of Tissot based on the mapometry of the cartographic grid [6]. In the case of distortion and transformation of digital cards, such a technique has significant limitations

Transformation, that is, the transformation of images into another system of coordinates in view of possible errors of source materials that arise under the influence of the nonlinear deformation of the base material (paper, photographic paper, plastic, etc.) and scanning errors, are the main operations in the compilation of maps.

To describe the quantitative relations of the parameters of the deformation field, we use the translation operator (displacement) in the tensor form on the plane:

$$T = \begin{vmatrix} \dfrac{\partial r_x}{\partial x} & \dfrac{\partial r_x}{\partial y} \\ \dfrac{\partial r_y}{\partial x} & \dfrac{\partial r_y}{\partial y} \end{vmatrix}. \quad (1)$$

Elements of the tensor T are the coefficients of linear relations.

$$dr_x = \frac{\partial r_x}{\partial x}dx + \frac{\partial r_x}{\partial y}dy;$$
$$dr_y = \frac{\partial r_y}{\partial x}dx + \frac{\partial r_y}{\partial y}dy, \quad (2)$$

which are projections of the displacement vector r on the coordinate

The coefficients of the linear correlations of relations (2) correspond to the coefficients of affine transformation 2D, so that

$$\frac{\partial r_x}{\partial x} = a_1; \frac{\partial r_x}{\partial y} = a_2; \frac{\partial r_y}{\partial x} = b_1; \frac{\partial r_y}{\partial y} = b_2. \quad (3)$$

Tensor T in accordance with (3) has the form:

$$T = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}. \quad (4)$$

Tensor T can be decomposed into a symmetric and skew-symmetric part [8]. The symmetric (deformative T_{def}) tensor has the form:

$$T_{def} = \begin{vmatrix} a_1 & \dfrac{1}{2}(a_2 + b_1) \\ \dfrac{1}{2}(a_2 + b_1) & b_2 \end{vmatrix}, \quad (5)$$

and skew-symmetric (tensor of rotation) correspondingly:

$$T_{rot} = \begin{vmatrix} 0 & -\dfrac{1}{2}(b_2 - a_1) \\ \dfrac{1}{2}(b_2 - a_1) & 0 \end{vmatrix}. \quad (6)$$

Own values T_{def} are from the solution of linear equations

$$\begin{vmatrix} a_1-\lambda_1 & \frac{1}{2}(a_2+b_1) \\ \frac{1}{2}(a_2+b_1) & b_2-\lambda_2 \end{vmatrix} = 0. \qquad (7)$$

Correspondingly

$$\lambda_1 = \frac{a_1+b_2}{2} + \frac{\sqrt{(a_1+b_2)^2+(a_2+b_1)^2-4a_1b_2}}{2};$$

$$\lambda_2 = \frac{a_1+b_2}{2} - \frac{\sqrt{(a_1+b_2)^2+(a_2+b_1)^2-4a_1b_2}}{2}, \qquad (8)$$

where λ_1 i λ_2 – eigenvalues of the symmetric tensor, which are the compression coefficients (tension) along the main axes of deformation and remain mutually orthogonal.

The orientation of the main axes of deformation is recognized by the angle *φ*, which is the angle between the direction of the main axis and the abscissa:

$$\mathrm{tg}2\varphi = \frac{a_2+b_1}{a_1-b_2}. \qquad (9)$$

The matrix T_{rot} is calculated the angle of rotation ω of the main axes of deformation:

$$\omega = \frac{1}{4}\left(b_1-a_2\right)^2. \qquad (10)$$

The indicated parameters are invariant characteristics of the quality of archival card materials.

The method of invariants is widely used in geodynamics [8, 5]. The essence of the method is in followed. Let's have some section with the catalog of coordinates x_i and y_i, increments *dx* and *dy*. The projections of each vector on the coordinate axis can be written in the form of a system of equations:

$$\begin{cases} u_i = e_{11}x_i + e_{12}y_i + a_1 \\ v_i = e_{21}x_i + e_{22}y_i + a_2 \end{cases}, \qquad (11)$$

where the value l_{ij} is a deformation tensor

$$T = \begin{vmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{vmatrix}. \qquad (12)$$

The equation (11) has six unknowns, what requires at least three such systems to be calculated.

So the simplest element-simplex in a two-dimensional space is the triangular, with the solution of which it is possible to obtain the maximum displacement *γ*, dilatation *ρ*, and the angle of rotation *ω* of the entire investigated area [8]:

$$\gamma_m = \sqrt{\gamma_1^2+\gamma_2^2};$$

$$\omega = \frac{1}{2}\left(e_{12}-e_{21}\right);$$

$$\rho = \frac{e_{11}+e_{22}}{2}, \qquad (13)$$

де $\gamma_1 = e_{11}-e_{22}$ і $\gamma_2 = e_{12}+e_{21}$.

In the case where the number of points is more than three, the solution can be carried out according to the MNC:

$$\begin{cases} u_1 = e_{11}x_1 + e_{12}y_1 + a_1 \\ v_1 = e_{21}x_1 + e_{22}y_1 + a_2 \\ u_2 = e_{11}x_2 + e_{12}y_2 + a_1 \\ v_2 = e_{21}x_2 + e_{22}y_2 + a_2 \\ u_3 = e_{11}x_3 + e_{12}y_3 + a_1 \\ v_3 = e_{21}x_3 + e_{22}y_3 + a_2 \end{cases}. \qquad (14)$$

For the studied triangle

$$C = \begin{vmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{vmatrix}; \; E = \begin{vmatrix} e_{11} \\ e_{12} \\ \lambda_1 \\ e_{21} \\ e_{22} \\ \lambda_2 \end{vmatrix}; \; A = \begin{vmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{vmatrix}, \qquad (15)$$

where *A* is a matrix that includes the numerical values of the first derivatives of the vector *C*:

$$C = \overline{A}E;$$
$$A^{-1}C = A^{-1}AE;$$
$$A^{-1}C = E. \qquad (16)$$

Table 1 shows the specific values of the deformation parameters for the three adjacent simplexes of the investigated cartographic material.

Table 1

Average values of deformation parameters

x_1, 10^{-7}	λ_2, 10^{-7}	γ_1	γ_2	γ_m	ω	ρ
1,0009	1,0003	12,4	–2,1	12,7	–6,3	21,4
1,0007	1,0005	–11,3	20,9	23,8	–5,4	–14,1
1,0028	1,0007	23,5	–24,5	35,4	–20,2	–18,7

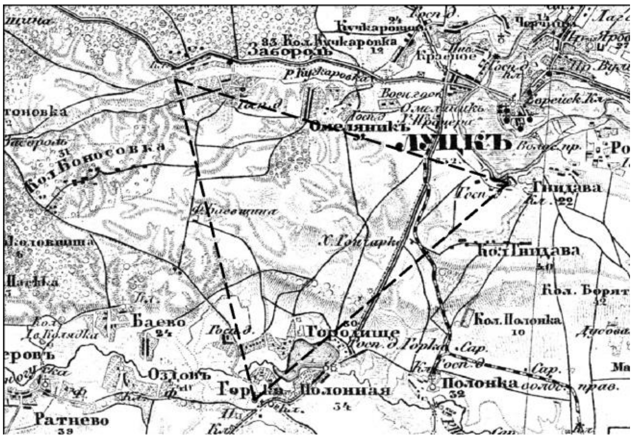

Fig. 1. Contour of the territory on the military topographic map of the Russian Empire in 1913 (1: 126 000)

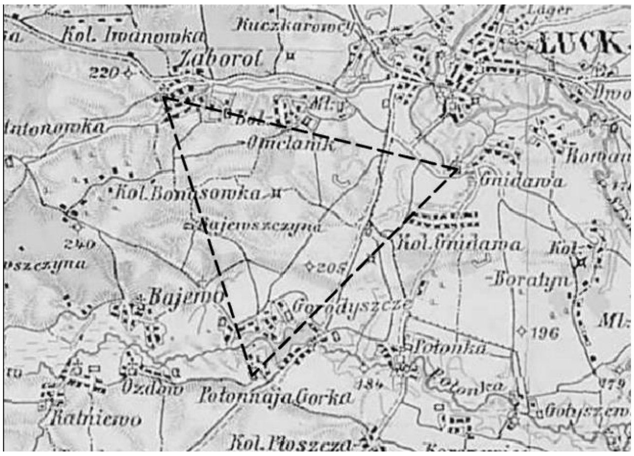

Fig. 2. Contour of the territory on the military military map of Austria-Hungary in 1910 (1: 200 000)

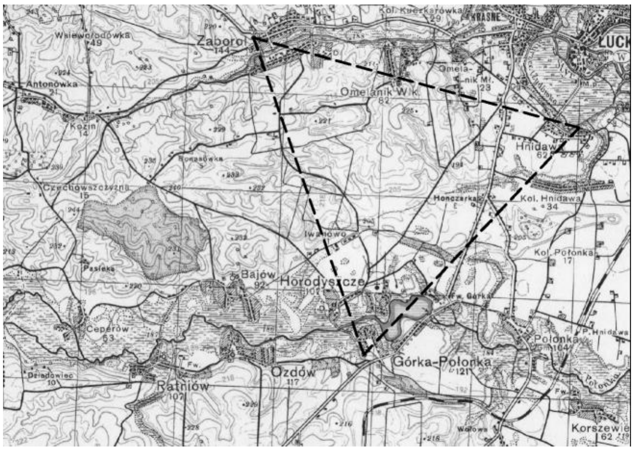

Fig. 3. Contour of the territory on the map of Poland 1925 (1: 100,000)

Similar results were obtained for 35 contours of the study area. It is established that 56% of situational elements of the maps are beyond the limits of accuracy, which is determined by modern normative documents.

2. Transformation of raster copies of archival top-maps. The research was carried out on raster copies of the original publication using the MapInfo software. Scanning was performed with a resolution of 600 dpi. The starting points were the angles of the trapezium frame, the points of its intersection with the lines of the kilometer net and the intersection of the kilometer net [4, 6]. This number of output points allows us to obtain an estimate of accuracy for different methods of transformation (affine and polynomial).

The main results of the study are as follows

2D conformal similar transformation is ineffective. The better results are obtained as a result of 2D affine transformation: $m_x = 106$ mkm, $m_y = 49$ mkm that allows you to increase the accuracy by 6 times. However, the errors m_x and m_y are significant.

Better results provide correction by polynomials of the second and third order. Polynes of the third order provided a correction with an accuracy of ± 46 microns and ± 29 microns, that is, they allowed to reduce the initial errors along the x-axis by 16 times and 6 times along the y-axis

3. Analysis of plans for solid (stable) contours. One of the possible methods for analyzing archival top materials is the creation of a PTC. Such the method was offered by the surveyors of the Moscow State University of Geodesy and Cartography. It allows to make more complete characteristics of the accuracy, the conformity of the raster image to the current state of the territory. However, such a technique involves the need for filtering gross and systematic errors. In the case of matching contours, it is expedient to use the mathematical apparatus of the mean-square prediction of Kolmogorov-Wiener [9, 10]. The essence of this method is as follows.

Let us have a set of random variables – the set l of points of the contour, obtained, for example, by updating topomaterials of a certain territory and the set of the same points *S* obtained from raster copies:

$$l = \left[l_1, l_2, \ldots l_g \right]^T; \quad (17)$$

$$S = \left[S_1, S_2, \ldots S_m \right]^T. \quad (18)$$

Then the linear estimate of the vector *S* will have the form:

$$\hat{S} = H \cdot l, \quad (19)$$

where *H* is a matrix of linear transformation of vector *S* into vector *l*, and the vector of errors is:

$$\varepsilon = \hat{S} - S. \quad (20)$$

Covariance vector matrix ε

$$C_{\varepsilon\varepsilon} = (\hat{S} - S) \cdot (\hat{S} - S)^T \quad (21)$$

is a covariance error matrix, and its diagonal elements are the dispersion of error σ^2.

Matrix $C_{\varepsilon\varepsilon}$ is a sum of two matrixes *A* and *B* [5]:

$$C_{\varepsilon\varepsilon} = A + B, \quad (22)$$

$$A = C_{ss} - C_{sl} \cdot C_{ll^1} \cdot C_{ls}, \quad (23)$$

where *A* is the degree of mutual mismatch of arrays *l* and *S*.

The main diagonal of this matrix contains the sum of squares of inconsistencies along the *x* and *y* axis. Illustrations of this approach will be performed according to data [5].

For analysis, there were used solid (stable) points of the fundamental building. Geodetic works were performed using electronic tacheometers. Shooting was performed at a scale of 1: 500.

If we combine the beginnings of the starting points for which the centers of weights of arrays *l* and *S* are taken, we will obtain:

$$l = \begin{pmatrix} -13,47 & -2,96 & 5,94 & 1,74 & 9,29 & 5,88 & -1,42 & -5,01 \\ 9,10 & -17,06 & -13,44 & -3,51 & -0,69 & 7,73 & 4,92 & 12,94 \end{pmatrix};$$

$$S = \begin{pmatrix} -14,36 & -3,24 & 4,91 & 1,43 & 10,97 & 7,42 & -1,80 & -5,34 \\ 9,07 & -16,96 & 13,69 & -4,00 & -0,22 & 8,26 & 4,92 & 12,60 \end{pmatrix}. \quad (24)$$

Then the matrix *A* will look like

$$A = \begin{vmatrix} 3,08 & 1,24 \\ 1,24 & 0,75 \end{vmatrix}. \quad (25)$$

The comparison of sets *S* and $\hat{S}$ the displacement of the points ΔS are determined.

$$\Delta S = \begin{pmatrix} 0,19 & 0,83 & 1,07 & 0,34 & 0,68 & 0,54 & 0,45 & 0,36 \\ 0,20 & 0,31 & 0,27 & 0,50 & 0,26 & 0,33 & 0,01 & 0,33 \end{pmatrix}. \quad (26)$$

By values ΔS we obtain an estimate f, which is different depending on the category of area. If we reject gross errors and repeat the analysis, we obtain:

$$f_1 = 0,24; 0,58; 0,47; 0,46; 0,55. \quad (27)$$

The values of f_1 do not differ significantly from f, which confirms the discrepancy between the contours of the raster and the plan. The obvious reason is the mistakes in the archival plan.

Conclusions

1. An original comprehensive methodology is offered, which allows to evaluate comprehensively the aging of archival top-maps, plans, etc. 2. Invariant tensor cartographic correctness factors are well known in mathematical cartography of the indicative Tissot. 3. It has been found that raster copies have significant errors. It is shown that the optimal model for their correction is polynomials of the second order and the third order. 4. The high efficiency in the rejection of gross errors by using the Kolmogorov-Wiener prediction has been proved.

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The invariants of PSC as criteria of cartographic correctness of archival topographic maps

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In the article there have been examined the issues of complex assessment of the quality of archival topographic maps, topographic plans, and others. The analysis of tensor factors of cartographic correctness of archival map materials has been made. The estimation of transformation of raster and original cartographic material has been executed. There has been made the conclusion about the expediency of applying the polynomial approximation of the second and third degrees. By correct topographic and geodetic data ther has been illustrated the stability of the plans of solid contours (PSC). There has been made the conclusion about the efficiency of Kolmogorov-Wiener filtration application.