UDC 528.04

TO DETERMINATION TOLERANCES FOR PRECISE GEOMETRIC LEVELING

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Problem formulation

Over the past decade, technology and instrumentation for geometric leveling has changed significantly. We began to use modern effective digital levelers with bar-coded rods, and the use of traditional level gauges significantly decreased. In some countries motorized geometric leveling is successfully implemented. Progressive satellite leveling is gradually becoming an alternative to geometric leveling. An instructional and normative base for defining the geometric leveling should be updated, but unfortunately the old instruction [1], which needs to be reviewed and updated, continues to operate.

In first place, this concerns an analysis of instruction requirements (or setting of necessary documents) for leveling rods (traditional and bar-code) in the current precise geometric leveling of III-IV classes. This article is devoted to this, and in the future it is planned to evaluate possibilities of devices that perform leveling, an influence and consideration of the environment on the result of leveling and alternative methods and methods for determining the excesses in the exact geometric leveling. There are grounds to assert that in aggregate this will lead to an increase in the accuracy of geometric leveling without carrying out additional works and can be used in preparing a new state modern geometric leveling instruction.

Presentation of the main material

Errors of geometric leveling are divided into: instrumental, caused by the influence of the external environment, and personal.

Due to the small distance between the rods, level surface of leveler, leveling points and reference level surface, one can take the arcs of concentric circles with a radius R=6380 km.

From Fig.1, write that

$$h_{AB} = a - b \ .$$

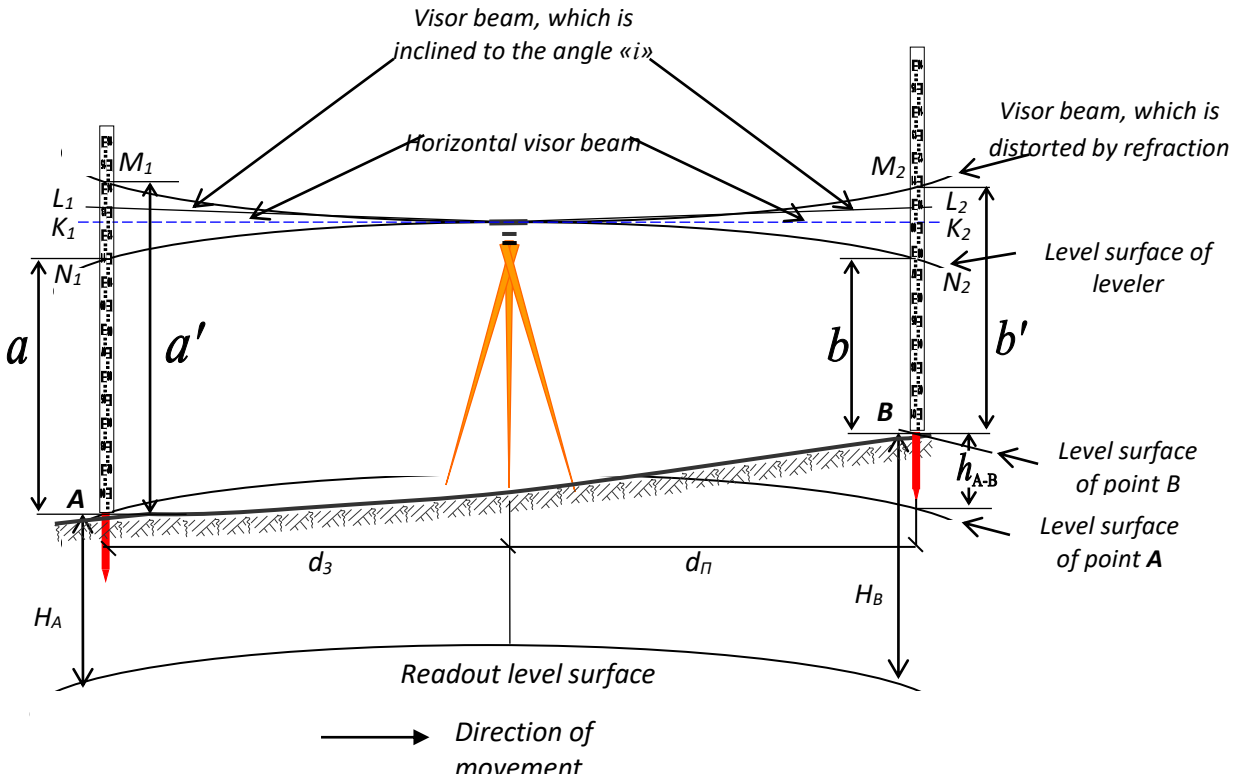

Fig. 1. Leveling from the middle

Line segments *a* and *b* are obtained from intersection of road with a level surface of leveler in points N_1 and N_2. In fact, the leveler sets the horizontal line and in the leveler will get points K_1 and K_2.

So, the line segments $K_1N_1 = q_1$ and $K_2N_2 = q_2$ there is nothing else, that influence of the Earth's curvature on read outing of rods, which are equal:

$$q_1 = \frac{d_З^2}{2R}, \quad q_2 = \frac{d_П^2}{2R}.$$

Excess is defined as the difference between the readouts, so the error will equal

$$\Delta q = \frac{1}{2R}\left(d_З^2 - d_П^2\right). \tag{1}$$

As we see from (1) error in excess of the Earth's curvature Δq is withdrawn, if distance to a rear rod $d_З$ is equal to the distance to front rod – $d_П$.

The actual visor beam of the leveler, usually non-horizontal, is located at an angle „i" to the horizon, and points of the rods L_1 and L_2 fall into the leveler. This will cause errors in rods readouts $\eta_1 = K_1L_1$ and $\eta_2 = K_2L_2$, which equals

$$\eta_1 = \frac{d_З \cdot i}{\rho}, \quad \eta_2 = \frac{d_П \cdot i}{\rho}.$$

In excess, this error will equal

$$\Delta\eta = \frac{i}{\rho}\left(d_З - d_П\right). \tag{2}$$

As we see from (2) error in excess of the angle «*i*», if it does not change during readouts of rear and front rods, is withdrawn, if distance to a rear rod $d_З$ is equal to the distance to front rod – $d_П$.

The ray that goes from the rod to the leveler, due to vertical refraction, propagates along the curve, and the point of the grid does not get point L_1 і L_2 but – M_1 і M_2, distorting the readouts also on values $r_1 = M_1L_1$ і $r_2 = M_2L_2$ (on Fig.1 is shown negative refraction $k < 0$, where k – vertical refraction coefficient)

$$r_1 = \frac{k_1 d_Ç^2}{2R}, \quad r_2 = \frac{k_{21} d_Ï^2}{2R}.$$

The error for the refraction in excess is calculated by the known formula

$$\Delta r = -\frac{1}{2R}(k_1 d_З^2 - k_2 d_П^2). \tag{3}$$

The excess error for vertical refraction will be partially reduced for minor excess if the leveler is located in the middle between the rods and the surfaces above which the rear and front rays pass through the same type.

As a result of these errors we will get rods readouts a' and b'. Readouts a' and b' are false on the values $M_1N_1 = q_1 + \eta_1 - r_1$ і $M_2N_2 = q_2 + \eta_2 - r_2$, respectively. Now, finally, we will get

$$h_{AB} = (a' - b') + (q_2 - q_1) + (\eta_2 - \eta_1) - (r_2 - r_1). \tag{4}$$

Analyzing formula (4), we see that in order to eliminate the errors in excess of the Earth's curvature and non-horizontal of visor beam, and also partly for refraction, the level should be placed in the middle between the rods. Which permissible inequalities of distance to the rear and front rods will be established further.

Let's consider the influence of main and other leveling errors, which adherence to permits will reduce their impact on the results of leveling.

Next, we will focus on the errors associated with leveling rods, tolerances on them and the methodology of leveling.

1. Error due to a non-perpendicularity of visor beam to the rod plane

Each of the rods will be set up perpendicular to the level surface and will not be perpendicular to the horizontal visor beam. The angle of non-perpendicularity ψ (Fig. 2) of the visor beam to the rod is calculated by the formula

$$\psi = \frac{d\rho}{R} \tag{5}$$

where *d* – distance to rod, which can be equal 150 m, R – radius of the Earth is equal 6380 km, ψ – central angle with a vertex in the center of the Earth, equal to the angle of non-perpendicularity of visor beam to rod, ρ"=206265. Hence,

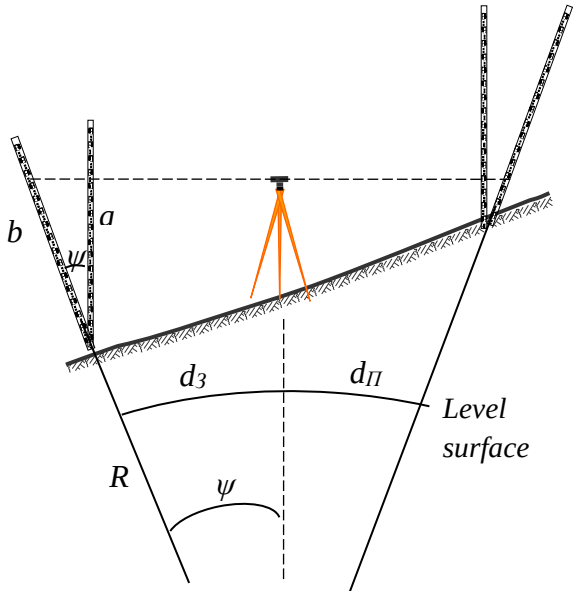

Fig. 2. To non-perpendicularity of rods to visor beam

$$\psi = \frac{150м}{6380000м} \cdot 206265'' = 4.8''.$$

Error Δh in excess will be the largest and will equal the difference between the rods readouts $\Delta h = a$, if excess h_{AB} on station will be the maximum. For III class $h_{AB} = 2.4\,m$. For such h error Δh the non-perpendicularity of the rod to the visor beam will be equal

$$\Delta h = b - a = b - cos\psi = 7 \cdot 10^{-7} mm \approx 0$$

on station. For move 60 km (2000 stations) with the maximum excess at the station, this error will be equal 0, 0012 mm. **Therefore, the error of non-perpendicularity of rods to the visor beam due to non-perpendicularity of the rod to the level surface will not affect the value of the determined excesses.**

2. Errors due to inaccurate set up of rods in a straight line position

The maximum **sections lengths** in geometric leveling reach to 5 km [1,2]. (Section is a move between neighboring rappers). Permissible residuals in such leveling moves of III and IV classes will be equal respectively 22 mm, $m_h = 11$мм and 45 mm, $m_h = 22\,mm$.

In Ukraine, the **move length** of III class leveling, usually does not exceed 60 km, and IV - 20 km [1,2]. Permissible residuals in such leveling moves of III and IV classes will be equal approximately 80 mm and 90 mm, but $m_h = 40\,mm$ and $m_h = 45\,mm$.

For mountain areas, the limit values for excess in the section can reach to 500 m. This is approximately equivalent to 250 stations of leveling in the section.

It is believed that for the shallow and flat areas the number of stations per kilometer of move is ten, and in the section for III and IV classes leveling – fifty [8]. The number of sections in the leveling move for III class is about 15, and for IV class – 5.

For **mountainous regions** of Ukraine, the number of stations in the leveling move for III and IV classes will not exceed 2000 and 1000, respectively. The highest mountain of Ukraine has a height of ≈2000 m. So, if go through it, the leveling move for III class there will be as much as 1,000 stations up and 1000 stations down.

For **shallow and flat terrain**, the number of stations in the leveling move for III and IV classes may reach 600 and 200 respectively.

In order the excess error determination practically did not affect its accuracy, it should be less than $0.33m_h$. **For a section** in move of III class it will equal 3.7 mm, and for IV class - 7.3 mm. Let's calculate a value of systematic error $m_{h_{\text{ст.сист}}}$ in excess of station. **In a mountainous terrain** for the III class, this error should be less

$$m_{h_{\text{ст.сист}}} \leq \frac{3{,}7\,mm}{250\,st} \leq 0{,}0148\,mm,$$

and for IV class ≤ 0,0292 *mm*.

Value of random error $m_{h_{\text{ст.вип}}}$ in excess on station for III class should be less than value

$$m_{h_{\text{ст.вип}}} \leq \frac{3{,}7\ mm}{\sqrt{250\ st}} = 0{,}2340\ mm,$$

and for IV class ≤ 0,4617 *mm*.

For shallow and flat terrain the value of systematic error $m_{h_{\text{ст.сист}}}$ in excess on station for III class should be less than value

$$m_{h_{\text{ст.сист}}} \leq \frac{3{,}7}{50} = 0{,}0740\ mm,$$

and for IV class ≤ 0,1460 *mm*.

Value of random error $m_{h_{\text{ст.вип}}}$ for III class should be less than value

$$m_{h_{\text{ст.вип}}} \leq \frac{3{,}7}{\sqrt{50st}} = 0{,}5232\ mm,$$

and for IV class ≤ 1,0324 *mm*.

For leveling move of III class it will equal 13 mm, and for IV - 15 mm. Let's calculate the value of systematic error $m_{h_{\text{ст.сист}}}$ in excess of station.

For mountainous terrain for leveling of III class, this error should be less

$$m_{h_{\text{ст.сист}}} \leq \frac{13\ mm}{2000\ st} = 0{,}0065\ mm,$$

and for IV class ≤ 0,015 *mm*.

Value of random error $m_{h_{\text{ст.вип}}}$ y in excess on station for III class should be less than value

$$m_{h_{\text{ст.вип}}} \leq \frac{13\ mm}{\sqrt{2000}\ st} = 0{,}2907\ mm,$$

and for IV class ≤ 0,4743 *mm*.

For shallow and flat terrain the value of systematic error in excess on station for III class should be less than value

$$m_{h_{\text{ст.сист}}} \leq \frac{13}{600} = 0{,}0217\ mm,$$

and for IV class ≤ 0,0750 *mm*.

Value of random error $m_{h_{\text{ст.вип}}}$ for leveling of III class should be less than

$$m_{h_{\text{ст.вип}}} \leq \frac{13\ \text{мм}}{\sqrt{600}\text{ст}} = 0{,}5307\text{мм},$$

and for IV class ≤ 1,0607*mm*.

As we see from the calculations, for leveling of III class, the values of random errors for section and move are the same. Similarly for IV class. The systematic errors for III class in move are in three times less than in section and for IV class – two or three times. Therefore, during further calculations of random errors at the station, they can be calculated both on section and on move. Systematic should be to calculate on move for both classes.

Inaccurate set up of rod has a systematic effect on the excess during the move. The horizontal visor beam of rod, which is located straight ahead, crosses on readout *a* (Fig. 3). If the rod is tilted to the angle α, then the rod readout will be equal to *b*

$$b = \frac{a}{\cos\alpha}. \qquad (6)$$

Difference Δb between a wrong readout *b* and correct *a* for **mountainous area** can reach

$\Delta b = b - a = 0.0037\ mm$, for III class, and

$\Delta b = b - a = 0.0125\ mm$, for IV class.

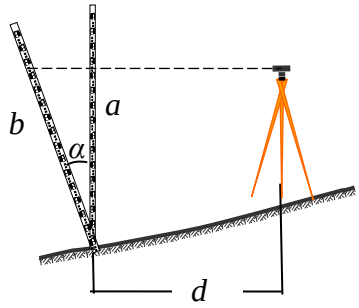

Fig. 3. To upright of rods setting up

Let's calculate from (6) the angle α for maximum rod readout $b = 2700 \ \ mm$.

$$\Delta b = 2700 - 2700 \cdot \cos\alpha = 0.0037 mm \text{ for III class, and}$$

$$\Delta b = 2700 - 2700 \cdot \cos\alpha = 0.0125 мм \text{ – for IV class.}$$

Here from $\alpha \approx 6'$ for III class, and $\alpha \approx 10'$ – for IV leveling class.

If we acceptances that the rod is possible to hold upright with an accuracy of 0,3τ, where τ is a value of level division, than **rod for leveling in the mountains must have a level with value of level division of 20′ for III class, and 30′ for IV class**. If the rod is set up in a straight line with an error of 10′ (rod has a standard 30′ level), then for 3500 stations, the error of exceed for leveling of III class can be equal to 40 mm.

For **shallow and flat terrain** we will have

$$\Delta b = b - a = 0{,}0217 \ mm \text{ for III class,}$$

$$\Delta b = b - a = 0{,}0750 \ mm \text{ for IV class.} \qquad (7)$$

Here from $\alpha \approx 14'$ from III class, and $\alpha \approx 25'$ for IV class. This means that rod level may have a value of level division for section 42′ for III class and 75′ for IV class. So, **for the leveling of III and IV classes in the shallow and flat terrain we can use rod with standard spherical levels with a value of level division of section 30′**.

There are several ways to determine the value of a spherical level division. An easiest, the value of a spherical level division, which is located directly on the rod, can be determined using an total station or roulette.

We install the rod near the wall by the middle of rod heel on a spherical point. Tilt the rod so that, for example, the left edge of the bubble touched the outer level division. With a sharpened pencil, we draw a bar on the wall near the 3-meter rod section. Tilt the rod so, that the bubble touches the inner edge of the level with the same left edge. Again there, draw a bar on the wall. Measure in millimeters the distance l between the shaded lines. Calculate the price of the level divisions

$$\tau' = \frac{l \text{ мм}}{3000 \text{ мм}} 3438'.$$

For example, l = 26.0 mm. Then τ = 29.8′. Perform the same definition for right edge of the bubble. Measurement accuracy l – 0,5 mm. The difference between two definitions should not exceed 1′. From the two definitions, the average is calculated.

If the value of level divisions is determined by a total station, then we set up the rod near the wall by the middle of the rod heel at the spherical point. Perpendicular to readout plane of the rod, at a distance of 5-10 m from it, set up a total station with or without a laser. If it is used a laser without a laser, then on a top of the rod attach a reflective film with a stylus bearing on it. Tilt the rod so, that, for example, the left edge of the bubble has touched the outer level division. Measure the distance S_1 to the mark in meters and simultaneously, weighing the horizontal circle A_1 on the vertical bar. Tilt the rod so, that the bubble touches the inner edge of the level with the same left edge. We again measure the distance S_2 to the mark and, simultaneously, weighing the horizontal circle A_2 on the same stroke of the mark. The value of level divisions in minutes will be equal

$$\tau = \frac{(S_1 + S_2)(A_1 - A_2)}{2 \times 3}$$

differences A_1-A_2 must be calculated in minutes.

For example, S_1 = 5,012 m, S_2 = 4,985 m, A_1 = 20°35′27″, A_2 = 20°17′35″.

$$\tau = \frac{(5{,}012 + 4{,}985)(20°35'27" - 20°17'35")}{2 \times 3} = 29{,}8'.$$

For the total station with a non-reflector function of measuring distance and angle, perform on the axis (edge of the divider) of the level rod. The measurement accuracy is 10 mm, and the angle is 5″.

If the distance between the bars (concentric circles) of a level ampoule is not equal to 2 mm, or there is only one bar, then a transparent material with concentric circles with a distance of

between 2 mm is affixed to the level ampoule. The center of these circles should be in the zero point.

Instructions [1] do not apply requirements to the value of a spherical level division. In the instruction [2] there is a requirement for the value of level division for III class leveling - 20′. For IV class there are no requirements.

3. Error in excess due to errors in meter and decimeter intervals

Based on the calculations in paragraph 2, we can assert that the error due to the inaccurate determination of the average length of one meter of the set of *rods* must not exceed the error in excess for the whole move of 13 mm for III class, and 15 mm for IV class. The incorrectly defined length of the meter range of the pair of rods will act as a systematic error in direct proportion to the excess.

In **mountainous area**, as can be seen from paragraph 2, the systematic error for III class move must not exceed 0, 0065 mm at the station. We assumed that the excess at a station in a mountainous area is two meters. Then errorm_a, the definition (not the length itself) for one meter length of the rods pair should be

$$m_a = \frac{0{,}0065\text{мм}}{2} = 0{,}00325\ mm.$$

and for IV class – 0, 0075 mm.

This accuracy can only be achieved by interferometers.

For shallow and flat terrain we have the following. Let the excess in the move of 50 m is concentrated, preferably in one section. Then the excess at station *h*, according to paragraph 2, will be equal

$$h = {}^{(50m)}/_{50_{st}} = 1\ m.$$

Consequently, the systematic error of one station will correspond to the error of the definition (not the length itself) of one meter length of rods pair. For III class, it should be less than 0, 0740 mm, and for IV class -0, 1460 mm. This accuracy can be achieved on MK-1 comparators, or interferometers.

As we can see, the error of one meter length of rods is difficult to remove because of high accuracy of determining the lengths of meter intervals.

Assume that the error in determining the intervals of rods must not exceed one third of permissible residual of move. Then, according to paragraph 2, we obtain errors $m_{h_{\text{ст.сист}}}$:

For a section, the III class leveling in the mountainous terrain (22mm:3=7,3mm)

$$m_{h_{\text{ст.сист}}} \le \frac{7{,}3\ mm}{250\ st} \le 0{,}03mm,$$

and for IV class $\le 0{,}06\ mm$.

For shallow and flat terrain

$$m_{h_{\text{ст.сист}}} \le \frac{7{,}3\ mm}{50\ st} \le 0{,}146\ mm,$$ for III class leveling

and for IV class $\le 0{,}30\ mm$.

For moves, III class leveling in mountainous terrain (80mm:3=26,7mm)

$$m_{h_{\text{ст.сист}}} \le \frac{26{,}7\ mm}{2000\ st} \le 0{,}013mm,$$

and for IV class $\le 0{,}030\ mm$.

Then, **for mountains area**, the length of a meter interval for III class leveling should be known with $m_a \le 0{,}007$ mm, and for IV class $m_a \le 0{,}015$ mm. This accuracy can be achieved on special MK-1 and others comparators.

For **shallow and flat terrain**, the length of a meter interval for III class leveling must be known $m_a \le 0{,}026$ mm, and the IV class $m_a \le 0{,}60$ mm.

For moves of III and IV classes with an excess of 50 m, wooden or fiberglass rails are used, the precision of determining the length of a meter intervals which reaches the control meter or topographical total station [3]. If the distance between the bars for a barcode rods is unknown, then

by means of the invariable rods [4]. In determining the lengths of a meter intervals by the control meter, the difference between the two measurements of the meter range is 0.1 mm. Measurement of each interval is performed directly and backward. All intervals are 12. Therefore, the measurement accuracy of one meter of a rods pair will be $0.1/\sqrt{2}/\sqrt{2}\times\sqrt{12} = 0.17$ mm. Limit error - 0.34 mm. Total station and invariable rods, mean square error determine the lengths of any intervals, approximately with the same accuracy.

In our opinion, it is incorrect to enter corrections in excess in the section for one meter length of the rods pair. Correctly adjust the correction to the counters. This, of course, requires more time for cameral calculations for optical-mechanical levelers. In digital levelers, such corrections can be entered directly into readout programs of the rods.

For example. For the first of rods, the length for both scales (black and red), the interval 0-10 - 1.0005 m, the interval 10-20 - 1.0005 m, the interval 20-30 - 0.9995 m. The average length of 1 meter of rods will be 1.000167. The second rod has the same error intervals. So the average length of 1 meter of the rods pair will be equal to 1.000167. Let the excess at the station be 2.0000 m. Then the readout, for reduced length of intervals for rear rod will be 1.9990, and for front - 0.0000. Thus, the measured excess at the station - 1.9990 m, and for rair will be 0.0000. Thus, the measured excess at the station is 1.9990 m, and the corrected will be $1.9990 \times 1.000167 = 1.999333$ m and is incorrect at 0.67 mm. Then the excess in the section will take 50 m, which was determined at 2-meter interval. Then the corrected excess would be $50 \times 1.000167 = 50.00833$ m. In reality, for this example, the corrected excess would be $50 \times 1.0005 = 50.025$ m. The difference is more than 16.6 mm. As we can see, this can lead to unacceptable inaccuracy in leveling move.

The requirement for instructions on the length of rods meter intervals is only relevant to the normalization of the decimeter intervals. Requirement instructions on the length of meter intervals of rods pair for III class - 0.5 mm and for IV-1 mm, can be justified as follows. If the error of the length of one meter is a 0.5 mm of rods pair. In applications - 0.4 mm. For IV class it is similar to 1 mm, in applications - 0.6 mm? Then, accepting the requirement of applications, for the III class, the error in the move of 600 stations will be $0.4\times\sqrt{600} = 10$ mm ≤ 13 mm. For IV class $0.6\times\sqrt{200} = 8$ mm ≤ 15 mm (see paragraph 2). In practice, the error of decimeter intervals for III and IV classes could be 0.5 and 1.0 mm respectively. Usually better than 0.4 and 0.6 mm, then there is a margin of accuracy for other errors that are difficult to take into account.

In our opinion, the stored sliding rods can be used in the III class, determining the lengths of meter intervals before and after the completion of the leveling move. Modern solid and stored rails, usually covered with fiberglass film, therefore, in our opinion, their length will change less, for example, due to changes in humidity.

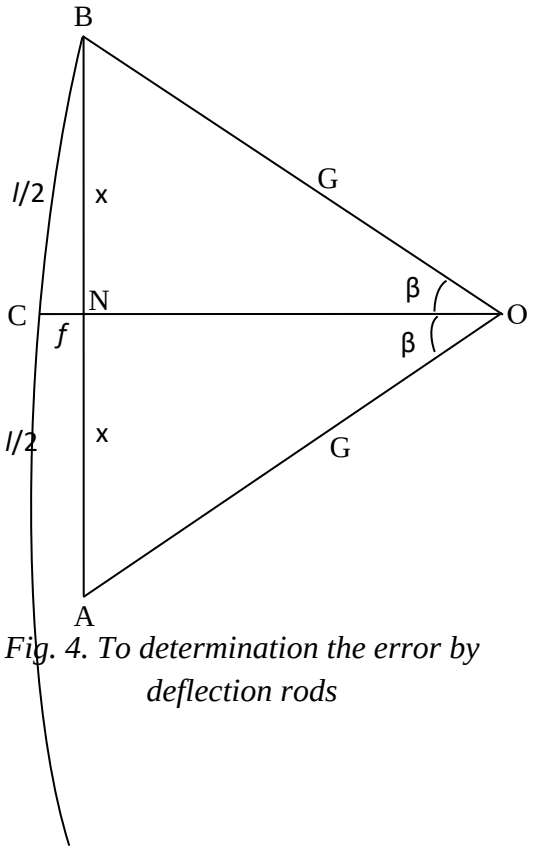

Fig. 4. To determination the error by deflection rods

4. Error in excess due to distortion of the rods

The instruction [1] does not limit the value of the arrow bend of the rod *f* (Fig. 4) for III and IV classes leveling. In the instruction [2], the arrow bend of the rod used in the leveling of III class must be less than 6 mm, and IV - 10 mm. We substantiate these requirements.

Let the rod be an arc of ACB = *l* radius G. The length of a chord ANB = 2*x*. So the difference between the lengths of the arc and the chord

$$\Delta l = l - 2x. \quad (9)$$

$$2x = 2G\sin\beta; \quad (10)$$

$$\beta = \frac{l}{2G}. \quad (11)$$

Let's decompose it out $\sin\alpha$ in a row and we will confine ourselves to two members of this row $\sin\beta = \beta - \frac{\beta^3}{6}$.

Substituting this value in (10), we will obtain

$$2x = 2G(\beta - \frac{\beta^3}{6}). \quad (12)$$

We substitute (11) into (12), and then (12) in (9), we will obtain

$$\Delta l = l - 2G(\frac{l}{2G} - \frac{l^3}{48G^3}) = \frac{l^3}{24G^2}. \quad (13)$$

From (13) we will find the radius *G*

$$G = \sqrt{\frac{l^3}{24\Delta l}}. \quad (14)$$

The considered excess error will depend on its value and is systematic. According to the calculations given above in paragraph 2 for the **mountainous area**, the error $m_{h_{\text{викр}}}$ in excess (in the large readout) should be less than 0.0065 mm for III class, and 0.015 mm for IV class. Respectively – 0,02 for the III class, and 0,08 for IV class.

The maximum excess at the station leveling can be equal to 2.4 m. If now, for simplicity of calculations, one of the readouts of rod is equal to 0.000, then the second rod readout is equal to 2.4 m, which include the above errors. For 3 m, this error for III class leveling is equal $m_{h_{\text{викр}}} = 3m \cdot \frac{0{,}0065mm}{2{,}4\ m} = 0{,}0081mm.$

So, **for mountainous area** the curvature radius of rod is

$$G = \sqrt{\frac{l^3}{24\Delta l}} = \sqrt{\frac{3000^3}{24 \times 0.0081}} = 372{,}10m,$$

for III class, and respectively 244,95 m – for IV class.

For shallow and flat terrain we will have G = 202, 26 m and G = 106, 07 m, respectively, for III and IV classes leveling.

Let's calculate for these data ***an angle β.*** For III class leveling **in mountain** we will have

$$\beta = \frac{l}{2G} = \frac{3}{2x372{,}1} \times 57{,}3^0 = 0{,}23^0,$$

and for IV class – $0{,}35^0$.

For shallow and flat terrain, we obtain $\beta = 0{,}42^0$ and $\beta = 0{,}81^0$, respectively, for III and IV classes leveling.

Let's calculate the permissible arrow bend *f* leveling rod and get

$$f = OC - ON = G - Gcos\beta = 416{,}02 - 416{,}02cos0{,}207^0 \leq 3{,}0\ mm$$

and $f \leq 4{,}6$ mm, respectively for III and IV classes **for mountainous area**, and $f = 5{,}4$ mm and $f \leq$ 10,6 mm, for III and IV classes **for shallow and flat terrain**. It can be argued that the instruction [2] gives a tolerance of $f \leq 6$ mm for III class leveling for shallow and flat terrain. For mountains, the tolerance in the instruction [2] is not given by *f*.

Based on carried out research we propose to establish the following tolerances on the arrow bend *f* of the rod for III and IV classes leveling:

- for the mountainous area $f_{III} \leq 3$ mm, $f_{IV} \leq 5$ mm;

- for the hilly and flat terrain of the $f_{III} \leq 6$ mm, $f_{IV} \leq 11$ mm.

5. Errors of excess due temperature expansion of the rods

Instruction [1] does not give recommendations on a introduction of corrections for the temperature of the rods. In instruction [2] the temperature during leveling should be measured at the beginning and at the end of the work (every day? in the middle of the day?). But how to enter the correction for the temperature is also not specified. In the examples of journal filling, the measured temperature is not shown.

This error occurs due to two factors:

1. The difference between temperature of rods comparatoring t_k and rods temperature $t_в$ during the measurements.

2. The difference between rods temperature during leveling at the station due to their uneven heating by the sun, and as a consequence a different rods temperature is comparable to rods temperature during the measurements.

Assume that the rods have the same temperature as temperature of rods comparatoring. Then the correct excess

$$h = a - b,$$

where *a* and *b* not distorted by temperature of the readouts.

If, for example, the temperature of the rods is the same, but not equal to as temperature of rods comparatoring, then readout *a*, if bigger, is distorted relative to that which was during the comparatoring more than the count *b*. So, in place of projection on the rod of visor beam, there will be another readout - *a'*. Similarly, instead of *b – b'*.

Consequently, the false excess due to the temperature expansion of the rods

$$h' = a' - b'.$$

The value of this error will be directly proportional to the value of the temperature difference of each of the rods and the temperature of the comparatoring and the value of excess move.

For example: rods was compared at + 15 ° C. During leveling, the temperature of the rods was the same and equaled + 5 ° C. The temperature difference is t_k - $t_в$ = 10°C. Let the readout *a′ = 2700 мм*, and the readout *b′ = 0300 мм.*

So false excess

$$h' = a' - b' = 2700 - 300 = 2400 мм.$$

Let's calculate the correct readout for wooden rods. The coefficient of thermal linear expansion of a tree is $5\times10^{-6}1/^0$C. Then the correct readout *a* = 2700 + (2700 × 10× 5×10^{-6}) = 2700, 14 mm. The correct readout *b* = 300 + (300 × 10° × 5×10^{-6}) = 300, 01 mm. The numbers will increase because the temperature is lower and the rods are reduced, and the correct excess of *h*, taking into account the temperature of the rods, is equal to

$$h = a - b = 2700{,}14 - 300{,}01 = 2400{,}13 мм.$$

The error in excess of 2.4 m is +0.13 mm. If the excess for the mountain area in the section 500 m, the error in the temperature of the rods, if not taken into account, will be 27 mm.

The coefficient of temperature expansion of sliding aluminum rods is $24\times10^{-6}1/^0$C, of composed fiberglass is $15\times10^{-6}1/^0$C. So, if you use aluminum rods, for the same calculations, the error in the move will be 121 mm. For fiberglass – 75 mm.

For move of **flat terrain** with an excess of 50 m, the error in the temperature of the rods will reach: for wooden - 2,7 mm, for aluminum - 12,1 mm and for fiberglass - 7,5 mm.

According to paragraph 2, the error of temperature influence m_{h_t} (so that it does not affect the excess in the section) should be less than 3.7 mm and 7.3 mm, for III and IV classes leveling, respectively.

Thus, the error of measuring the temperature of rods $\pm10^0$ for mountainous terrain is inadmissible, and for moves with an excess of 50 m it is permissible for wooden and fiberglass rods for the IV class.

In our opinion, the temperature of the rods, provided that the excess error in the section through the temperature of the rods m_{h_t} exceeded 3.7 mm for III class, it is necessary to know with accuracy:

a) the mountainous terrain for wooden ≈ $1{,}3^0$; fiberglass ≈ $0{,}5^0$, and for aluminum ≈$0{,}3^0$, for class IV $2{,}5^0$, $0{,}9^0$ and $0{,}5^0$ for wooden, fiberglass and aluminum rods, respectively. Therefore, instruction [2] recommends in the mountains for III class leveling use an inverted rod, the temperature of which you need to know with an error ≈14^0. Even the use of wooden rods is problematic for the IV class, and it is practically impossible to use fiberglass and

aluminum rods. The temperature of the wooden rods must be measured at each station and entered in the correction in excess;

b) shallow and flat terrain for wooden rods ≈3^0; fiberglass ≈5^0, and for aluminum ≈3^0; for IV class - 25^0, 9^0 and 5^0 for wooden, fiberglass and aluminum rods, respectively. Consequently, for III class, it is possible to measure the air temperature through several stations and introduce excessive corrections for it, or if there are no sharp changes in temperature, it can be obtained using specialized mobile applications.

The name of the fiberglass rods does not entirely correspond to reality, because it is wooden rod covered with fiberglass film.

Conclusions

The detailed researches of leveling rods has been carried out and obtained results allowed to critically evaluate and substantiate correct the tolerances (discussed in item 1-5) to sources of errors in precise geometric leveling associated with the use of traditional and bar code rods. The obtained tolerances and recommendations should be used in the new state manual on geometric leveling.

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To determination tolerances for precise geometric leveling

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The influence of geometric leveling errors connected with rods is investigated. The requirements for the upright of rods setting up, adding corrections for the length of one meter of a pair of rods, the value of deflection rods, are justified, and shown that it is necessary to add corrections for the temperature of rods, and so on. The fulfillment of these requirements will promote the accuracy and efficiency of III and IV classes leveling without additional costs.