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ADAPTIVE METHODS FOR EVALUATION AND INTERPRETATION OF EARTH'S DEFORMATION FIELDS USING GNSS DATA

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This article discusses the prospects for the creation of models and methods for evaluation of deformation fields of the Earth that are adequate to the data of physical surface monitoring by the GNSS method. Existing approaches and research methods that use the GNSS data are analyzed. The insufficient degree of conformity of methods to the potential informative resource of these data was revealed. Requirements for models of deformation fields and new adaptive methods of GNSS data processing were formulated. An alternative generalizing approach and methods of deformation analysis are proposed. They are based on the theory of differential representation of transformations of Riemannian space images in the form of complex and elementary diffeomorphic manifolds.
Keywords: GNSS, deformation analysis, Riemannian diffeomorphic manifolds, homeomorphism, metric tensor

Statement of the problem

The evaluation and analysis of deformation fields of the Earth is among the priority problems of modern geodynamics. The solution of the problem is achieved by the efforts of various natural sciences. Among them, the geodetic branch of knowledge occupies a special place. Geodetic monitoring is able to quantify the modern deformation processes of different scales by re-observing of geodetic networks, processing and interpretation of the results. The objectives and content of researches in this direction are determined by resolutions of the International Association of Geodesy within the framework of activities of Sub-commission 3.2 "Crystal Deformation" of Commission 3 "Earth Rotation and Geodynamics". The theoretical basis of researches is the mathematical theory of elasticity. At the present stage of researches, the main source of input data is the results of monitoring of coordinates, which are determined by the method of GNSS (Global Navigation Satellite System) [http://iag.dgfi.tum.de/fileadmin/handbook_2012/333_Commission_3.pdf].

The active development and introduction into the research practice of the GNSS method of Earth's monitoring revealed new perspectives for solving the problems of geodynamics. The use of GNSS data significantly increased the potential of geodetic monitoring of deformation fields. At the same time, this caused problems that are related to rethinking of theoretical foundations of deformation analysis and introduction of new field models and data processing methods.

Analysis of the research and unresolved parts of the general problem

The evolution of geodetic methods for solving the problem has been successfully disclosed by Professor A. Dermanis in his review article [Dermanis, 2009]. Own analysis of methods of the deformation analysis in geodynamics is given in articles [Tadyeyev, 2015b, 2016b]. The main content of methods and some examples of typical studies are systematized in the comparative table 1. Now we will not fully disclose the essence of methods. But note some of their features, which are fundamentally important in terms of own vision of prospects for solving the problem using GNSS data.

All methods of evaluation of deformation fields by geodetic data are traditionally based on the mathematical theory of elasticity. This approach was introduced into the research practice at the beginning of the last century. For objective reasons, such a powerful theoretical basis has found practical application in the simplest form of a linearly homogeneous model of the infinitely small deformation of the continuous medium.

Initially, the model was realized by the finite element method only on two-dimensional simplexes in a flat rectangular coordinate system with the application area in the evaluation of the horizontal component of deformation fields. On the basis of the linear functional model of the displacement of points at the vertices of the simplex, an analytic solution of the problem was achieved. The ultimate goal was to construct a deformation tensor as the main geometric object of the deformation process. Tensor opened the opportunity to interpret the phenomenon by different in content numerical characteristics. This forced choice was caused by the need to use the results of repeated observation of classical horizontal geodetic networks as input data for solving the problem.

Subsequently, the forms of geometric and corresponding functional models are being improved. As basic, various nonlinear functions are used, and solutions are achieved by numerical methods. At the same time, the solution of tasks in the direction of using the input data in two-dimensional curvilinear coordinate systems, such as spherical and ellipsoidal, are also improved. Comparatively with a plane, the model surfaces corresponding to these systems are more realistic in terms of the Earth's physical (topographic) surface mapping. Due to this, the best adequacy of the tensor and

deformation fields modeled on these surfaces was achieved in the context of the interpretation of processes not only local, but also regional scales.

However, the theoretical basis and the used mathematical model remain unchanged. For this reason, at the stage of formation of the tensor, the basic nonlinear functions are subject to linearization, and in the end he expresses only linear regularities of the deformation. This is a consequence of the use of a mathematical model that can transfer only the simplest affine transformation of coordinates of finite elements vertices. In addition, the formal, mostly, division of the territory into finite elements predetermines the risks of loss of the adequacy of the linear model to the real deformation. As a result, this causes the doubtful reliability of data processing results and the phenomenon interpretation.

Table 1

Methods for evaluation of deformation fields on the basis of mathematical theory of elasticity

Components of fields	horizontal (two-dimensional)				spatial (three-dimensional)		
Theoretical basis	mathematical theory of elasticity						
Mathematical model	linear-homogeneous model of infinitely small deformation of a continuous medium						
Method of research	finite element method						
Modeling surface	plane		geosphere	earths ellipsoid	geosphere	earths ellipsoid	topographical
Coordinate system	flat rectangular x, y		spherical φ, λ	geodetic ellipsoidal B, L	local spherical φ, λ, r	geodetic ellipsoidal B, L, H	rectangular geocentric x, y, z
Geometric model	simplex (triangle)	arbitrary forms	quadrangle	simplex (triangle)	arbitrary forms	quadrangle	simplex (tetrahedron)
Geometric object	Euler-Lagrange two-dimensional tensor of the first kind				Euler-Lagrange two-dimensional tensor of the first kind + tensor of rotation	Euler-Lagrange two-dimensional tensor of the first kind + tensor of rotation + Euler-Lagrange tensor of the second kind	Euler-Lagrange three-dimensional tensor of the first kind
Functional model	linear function of two variables	arbitrary function of two variables		linear function of two variables	arbitrary function of two variables		linear function of three variables
Basis of tensor formation	affine transformations of coordinates, which predetermine the linearization of the functional model						
Scales of fields	local	regional, local					local
Some examples of typical studies	[Harada, Shimura, 1979; Bibbi, 1982]	[Schneider, 1982; Pietrantonio, Riguzzi, 2004]	[Marchenko et al., 2012]	[Voosoghi, 2000; Grafarend, Voosoghi, 2003]	[Savage et al., 2001; Kreemer et al., 2009]	[Altiner et al., 2006; Grafarend, 2012]	[Brunner, 1979; Kiamehr, Sjöberg, 2005]

Introduction into the research practice of the GNSS method of coordinate monitoring prompted the development of methods for evaluation of three-dimensional deformation fields. However, most methods do not foresee the direct use of coordinates in the spatial geocentric system ITRS, in which they are monitored using the GNSS method. Instead, their transformation into a spherical or ellipsoidal system is pre-executed. As a result, numerical estimates of deformation fields belong to two-dimensional model surfaces that are parameterized by the used coordinate systems and are only embedded in the three-dimensional space. This property of methods is a consequence of the use of the theory of plates and shells within a limited framework of the finite element method. This method forces to consider the model surfaces as graded and causes their division into triangles (or quadrilaterals in the form of a grid). If the division of territories into finite elements was somehow justified as a consequence of the use of observations in classical horizontal networks, the use of such an approach for the processing of data in modern networks of permanent GNSS stations is devoid of any logical content, since no division into elements of any geometric forms in this case does not exist. But even so, then the two-dimensional tensors of the first kind, which belong to one or another surface, are subject to evaluation, and the interpretation of deformation fields is carried out only in the horizontal component part. Evaluation of vertical deformations is achieved using the third coordinate as a consequence of determining the invariants of a tensor of the second kind or associated with the first a rotation tensor. Consequently, the evaluation of the horizontal and vertical components of the deformation fields is realized independently. However, the authors of the research convinced that by their joint interpretation such an approach can provide the study of spatial deformations of the topographic surface from the point of view that the surface is considered as embedded in the three-dimensional space of the ITRS system.

The evaluation of spatial deformations by direct use of coordinates in the ITRS system is achieved within the framework of a three-dimensional simplex - tetrahedron. This model was previously used in research practice, if it was possible to combine the points of classical horizontal and levelling geodetic networks. In this approach, the three-dimensional Euler-Lagrange tensors of the first kind are evaluated. But then the interpretation of the deformation fields is carried out by invariants of the tensor, which belong not to the topographic surface, but to the barytsenters of the simplexes. Consequently, the upper horizons of the Earth's crust are subject to evaluation. In terms of studying the genesis and interpretation of geodynamic phenomena in general, this property of the method has an obvious positive. But, taking into account that the coordinates of the vertices of the simplexes belong to the topographic surface, this method has a limited practical application as such that is suitable for the evaluation of local deformations only in conditions of a terrain with the broken relief.

Formulation of the problem

Creation of adaptive methods for evaluation of the deformation fields of the Earth using GNSS data.

The main material of researches

Requirements for adaptive methods

Analysis of the used methods shows an insufficient degree of their conformity to the potential informative resource, which for the solution of the problem have the results of continuous monitoring of coordinates of the Earth's physical surface by the GNSS method. Taking into account the widespread introduction of GNSS data into research practice, this indicates the need for further improvement of methods for their processing. Improvements should be based on such theoretical approaches that the adequacy of models of deformation fields, corresponding adaptive methods of data processing and analysis of the obtained results was ensured in relation to the input data. Adaptive methods for evaluation of deformation fields should satisfy, at least, the following requirements:

- the ability to directly use GNSS data in a three-dimensional geocentric system;
- the possibility of evaluation, first of all, three-dimensional deformation fields;
- determination of deformation estimates that belong directly to the topographic surface, which shows deformation processes and is subject to direct monitoring by the GNSS method, but not to any model surfaces;
- no dependence on the division of territory into finite elements;
- evaluation of deformation fields not only on local or regional scale, but also on a planetary scale;
- the possibility of transfer of nonlinear deformation effects by a functional model of the displacement of observation stations and a tensor which is not burdened by the condition of linearization of the basic functions of this model.

Generalizing theoretical basis

Substantiation of the choice of the generalizing approach and the results of the creation of methods that can satisfy the formulated requirements presented in the articles [Tadyeyev, 2015b, 2016a, 2016b, 2017]. The main content of methods is shown in the comparative table 2. The basis of the methods is the theory of the differential representation of transformations of images of the Riemannian space in the form of complex and elementary diffeomorphic manifolds. To achieve the solutions of the problems of deformation analysis on this basis, methods of the projective differential geometry

and methods of describing the changes of the Riemannian metric in the tangent space were used.

The application of such a theoretical basis for solving the problem is possible by the hypothesis that the transformation of an arbitrary three-dimensional or two-dimensional closed and continuous domain of the space has a geophysical origin. If as a domain of space to consider the topographic surface, then its spatial-temporal transformations are identified with the deformation. The topographic surface, as an object of deformation analysis on such a basis, is not limited to its size or geometric form. As a result, the limitation on the scale of the surface is removed and the need for its division into finite elements is eliminated.

Using, as a complex manifold, the tangent Euclidean space, which is parameterized by the Cartesian coordinate system, opens up the perspective of describing the deformation fields on all scales, including the global (planetary) scale. Cartesian parameterization allows direct use of input data in the geocentric rectangular ITRS system, because it is a partial case of Cartesian, or in any three-dimensional curvilinear system, for example, ellipsoidal or spherical. At any choice, then directly topographic surface is modeled, regardless of its scale. The use of elementary manifolds makes it possible to evaluate the horizontal component of deformation fields on the local and regional scales. Elementary manifolds are considered in the form of a tangent plane or arbitrary curvilinear surfaces with corresponding two-dimensional parameterizations. In this case, the deformation processes are evaluated in relation to those or other model surfaces, as it is realized by existing methods with the previous transformation of coordinates from the ITRS system.

The general theory of transformations (mappings) of images of the Riemannian space allows using the property of a homeomorphism of diffeomorphic manifolds. If the manifolds have the same dimension n and undergo a mutually unambiguous and continuously differentiated transformation of $n-1$ class, then they respectively define the class of basic functions that convey this transformation. Then it is smooth or piecewise-smooth functions of C^{n-1} class. Consequently, the functional model acquires only restrictions on the uniqueness, continuity and differentiation of its basic functions. But do not limit the analytical forms of functions. In terms of the problem to be solved, it enables to convey the transformations by nonlinear functions, which are empirically established by displacements of observation stations along a certain parametrically determined curve only with the condition of ensuring a sufficiently smooth change of the Riemannian metric. Thus, the perspective of the transfer of nonlinear regularities of the deformation is provided by the main geometric object - a bivalent covariant metric tensor. In this approach, the formation of a tensor is not burdened by the condition of linearization of basic functions. The tensor is formed by the metric form of the transformed (deformed) domain of the space, in particular, by the square of the length of the linear element, which is expressed in the differentials of coordinates of the transformation domain (before deformation), taking into account the complete differentials of the homeomorphic basic functions.

Metric forms of the space domain these are the linear elements of the measure, which describe not only the displayed domain but also the internal geometry of the transformation of space as a result of changing its metric properties. Consequently, if according to the accepted hypothesis to admit the geophysical nature of transformations, then homeomorphic functional model and formed on its basis metric forms and metric tensor are able to transfer the change of metric properties of the domain by numerical characteristics of the different geometric content. In fact, these are the parameters of the deformation of the domain of space, as an object of study, in the generally accepted interpretation of the deformation analysis in geodynamics.

Table 2

Methods for evaluation of deformation fields on the basis of projective differential geometry

Components of fields	spatial (three-dimensional $n = 3$)			horizontal (two-dimensional $n = 2$)		
Theoretical basis	projective differential geometry in the environment of the Riemannian space					
Mathematical model	differential representation of transformations of Riemannian space images					
Methods of research	methods of projective differential geometry and methods of describing the changes of the Riemannian metric in the tangent space					
Fundamental hypothesis	transformations of space has a geophysical origin					
Forms of images	complex diffeomorphic manifolds with dimensionality $n = 3$			elementary diffeomorphic manifolds with dimensionality $n = 2$		
	tangent Euclidean space			tangent plane	tangent curvilinear surfaces	
Systems of parameterizations (coordinates) of manifolds	rectangular geocentric x, y, z	geodetic ellipsoidal B, L, H	spherical φ, λ, r	flat rectangular x, y	geodetic ellipsoidal B, L	spherical φ, λ
Modeling surface	topographical			plane	earths ellipsoid	geosphere
Geometric model	arbitrary closed and continuous domain of space					
Geometric object	bivalent covariant metric tensor of the transformation (deformation) of the space domain					
	three-dimensional			two-dimensional		
Functional model	arbitrary smooth or piecewise smooth homeomorphic functions of the C^{n-1} class					
Conditions of realization of the functional model	providing a sufficiently smooth change of the Riemannian metric					
Basis of a tensor formation	change of the main quadratic (metric) form of the space domain on the basis of a homeomorphic functional model					
Scales of fields	global, regional, local			regional, local		

The described theoretical approach to solving the problems of deformation analysis has a generalizing character in relation to the one used in modern studies, even in terms of the theory of tensor analysis and its application in geometry, mechanics and physics: the linearly homogeneous and infinitesimal deformation of the continuous medium is considered as a transformation in the environment of the affine space, which is trivial with respect to the Euclidean and Riemannian. This fact is evidenced by many classical publications in this area, for example [Rashevskij, 1967; Sokolnikoff, 1971], etc.

Results of solutions

Using methods of the projective differential geometry and techniques for describing changes of the Riemannian metric in the tangent space, on the generalizing theoretical basis, solutions were made in typical geodetic three- and two-dimensional coordinate systems, and the working formulas were obtained for calculating the deformation characteristics of the space domain. The formulas are suitable for expressing the regularities of deformation of any character that can transfer the empirical homeomorphic functional model. Characteristics are expressed in components of a bivalent covariant metric tensor. Taking into account the established practice of deformation analysis in geodynamics, the different in content characteristics are divided into three groups: 1) the main linear deformations as indicators of the change of form in a given direction; 2) the dilatation as an indicator of the change in the volume of the Earth or the area of part of its surface with the preservation of the general form (scale factor); 3) indicators of angular distortions. The analytical expression of characteristics of all three groups was carried out in two directions.

1. Three-dimensional deformation fields

In the part of the expression of the first group of parameters there are derived formulas for calculating the coefficients of elongation, compression and shear of the topographic surface. The directions of these indicators are determined in the geocentric polar system. Formulas for calculating of characteristics in any given direction, along directions of coordinate axes, in projections on coordinate planes, as well for the triad of extreme values with corresponding spatial orientation are obtained. Solutions are adapted to direct use of input data in the ITRS system [Tadyeyev, 2016b].

The formulas for calculating the coefficients of absolute and relative volumetric extensions as indicators of the dilatation of the topographic surface of the Earth on a global scale, as well as the coefficients of the change in the surface area of the regional or local scales, are derived. The evaluation of the dilatation fields is also achieved by direct use of the rectangular spatial coordinates of the GNSS stations. Furthermore, the use of coordinates in any three-dimensional curvilinear system is allowed. Then the algorithm for their transformation from the ITRS system is embedded into the structure of the metric tensor [Tadyeyev, 2017].

In the part of the expression of a group of parameters of angular distortions, the formulas for calculating the angles of rigid rotation in projections on the coordinate planes of the ITRS system as part of the global spatial deformations of the Earth are derived. This problem is considered in the interconnection with the probable current deformations of the ITRS system. This hypothesis is substantiated by the geophysical content of the concept of the ITRS creation as a consequence of the violation of the NNR-condition during anomalous global tectonic processes. It is proved that, under the condition of sufficient significance, the angles of rigid rotation of the Earth are indicators of angular distortions of the ITRS system. If they are expressed in deviations from the orthogonality of the axes in the ITRF-realization, then we obtain the measures of the oblique Cartesian system at any time of observation after realization. The truthfulness of the hypothesis of the current deformation of the coordinate system can be verified by comparing the empirical values of the rotational angles (along with the indicator of global dilation as a scale factor) with their official analogues provided by the ITRS Center of the IERS (International Earth Rotation Service), for example, with regard to the latest ITRF2014 implementation [Altamini et al., 2016]. It should be noted that the official data is obtained by the combined processing of results of Earth's monitoring using the satellite geodesy methods GNSS, VLBI, SLR, and DORIS on the basis of the well-known Helmert's transformation in its linearized form [IERS Conventions, 2010]. Own obtained results allow monitoring the current deformations of the system only by GNSS data and take into account the perspective of expression of nonlinear deformation regularities. In this part of the research we also obtained analytical expressions for the directions of the axes of the deformed system. The recommendations for formulating and solving the tasks of deformation analysis in the case of empirical confirmation of the hypothesis of the coordinate system distortions are also presented [Tadyeyev, 2016a].

In the context of this rubric, the obtained results can be used to solve the problems of modern geodesy in the interconnection with geodynamics in the study of reference coordinate systems.

2. Two-dimensional deformation fields

In this direction, the problem is considered from the standpoint of the differential representation of transformations of images of the Riemannian space in the form of elementary diffeomorphic manifolds such as tangent surfaces of arbitrary (including zero) curvature. In terms of the classical theory of transformations of projective-differential geometry, this problem is interpreted as a partial case in relation to the previous one and is more often formulated as the theory of surfaces mappings. The simplest solution is provided by the theory of surfaces in the Euclidean space. If anyhow parameterized two-dimensional surface is embedded in a three-dimensional Euclidean space, then it must be regarded as a two-dimensional Riemannian space with an appropriate metric quadratic form and a metric tensor. Then the coefficients of the tensor will take into account the interconnections of the curvilinear coordinates, which parameterized the surface, and the spatial coordinates of the tangent (in the Riemannian) Euclidean space. From the point of view of the solution of the task, this allows to directly use the spatial geocentric coordinates of the observation stations in the ITRS system. Their transformation into a two-dimensional surface system will be embedded in the structure of the tensor. Although the tensor field with the corresponding deformation components will not relate directly to the topographic, but to the chosen curvilinear surface, however, the possibility of simultaneous modeling of deformation fields of different geographic locations in a single geocentric system is open. The use of such a theoretical basis makes it possible to evaluate the horizontal component of deformation fields on a regional or local scale.

As a result of solving the problem on such basis, we obtain the working formulas for the expression of all three groups of deformation characteristics that belong to the typical geodetic two-dimensional model surfaces with the corresponding parameterization systems. In the article [Tadyeyev, 2013b] the results of the solution in a spherical geocentric system are presented, in the article [Tadyeyev, 2015a] - in the geodetic (ellipsoidal). The prospects for transforming the deformation field from one curvilinear surface to another are also revealed here. To evaluate the two-dimensional deformations on a local scale, it is enough to use the results of the solution of the problem on a plane in a rectangular system. Such results are presented in the article [Tadyeyev, 2013a].

Although these solutions are independently obtained, they can not claim to primacy and scientific novelty, since similar ones are already used in research practice. For example, authors [Savage et al., 2001] and other followers in their research use a local spherical coordinate system. Researches on authorship [Grafarend, Voosoghi, 2003] and others of the same type based on the use of an ellipsoidal system. However, own results are presented in a generalized and unified form and capable of transmitting the deformations of any nature. Other typical approaches at the stage of formation of the tensor require the linearization of the basic functions of coordinates. From this perspective, the own obtained results have undoubted practical significance.

Conclusions

The analysis of approaches and methods for evaluation of Earth's deformations by geodetic data showed that the used models of deformation fields and methods of their evaluation have the insufficient degree of conformity to the potential informative resource of GNSS coordinate monitoring data. Based on this,

requirements for models and adaptive methods for evaluation of deformation fields by GNSS data were formulated. An alternative generalizing approach and methods of the deformation analysis are proposed. As a theoretical basis, the theory of differential representation of transformations of images of the Riemannian space in the form of complex and elementary diffeomorphic manifolds was used. Methods of the projective differential (metric) geometry and techniques for describing the changes of the Riemannian metric in the tangent space were used to achieve the solutions.

On this basis, in the typical geodetic coordinate systems, solutions were made and the working formulas were derived for calculating of various numerical characteristics of the main linear deformations, dilatation and angular distortions. The results of solutions in three-dimensional systems make it possible to evaluate the deformations directly on the topographic surface in the global, regional and local scales. In the case of a global scale, the solutions are considered in the interconnection with the probable current deformations of the ITRS system, in which GNSS coordinate monitoring, is carried out. The formulas take into account these deformations during the deformation analysis. The results of solutions in two-dimensional coordinate systems allow evaluating the deformations on a regional and local scale with an interpretation relative to the model surfaces corresponding to these coordinate systems. In all working formulas, the deformation characteristics are expressed by the components of a bivalent covariant metric tensor, which is not burdened by the condition of linearization of the basic functions and, therefore, provides the perspective of the transmission of nonlinear deformation effects.

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