METHOD OF DETERMINING TROPOSPHERIC PARAMETERS FROM COMPATIBLE DATA OF SLR AND GNSS OBSERVATIONS

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**Formulation of the problem**
Requirements for accuracy of geodetic parameters determination are constantly increasing with the development of applied and fundamental research in various areas of astronomy, geodynamics, geophysics, as well as for many practical applications in geodesy, astronautics, and studies of natural disasters and in climatology. At the present stage, studies of the Earth system are defined as combinations of many global and regional solutions obtained through various methods of space geodesy. In turn, these methods are sensitive to various global parameters of the Earth. Their combination allows for obtaining new data on phenomena and processes, provided from common solution that takes into account all available observations of each of the individual methods.

**Analysis of recent research and publications related to solving this problem**
During the last few decades, many works have been published, in which the problem of a combination of various methods of space geodesy, in particular [1-3], is explored. Separately, such information is used to improve the accuracy of realization of terrestrial and celestial coordinate systems, estimates of Earth’s rotation parameters, obtain observation data for the analysis of changes in geodynamic parameters, studying and monitoring of processes occurring in the troposphere and the ionosphere of the Earth, studying the motion of lithospheric plates, etc.

**Exposition the main material**
Space geodesy is a section of geodesic science that is used to study the Earth's surface, its deformation, orientation, gravitational field, as well as atmosphere, observes the artificial and natural satellites of the Earth and interferometric observations of extragalactic objects [4].

The main tasks of space geodesy are:
- high-precision determination of geometric three-dimensional coordinates and velocities (in global, regional and local reference systems);
- definition of the gravitational field of the Earth and its temporal variations;
- modeling of geodynamic phenomena (motion of tectonic plates, deformation of the Earth’s crust), including parameters of the Earth's rotation (motion of poles, rotation of the Earth, precession and nutation).

At the same time, the main condition for solving these problems is the presence of a common reference system in relation to which necessary measurements are performed.

In addition, space geodesy contributes significantly to meteorology by studying the effect of the atmosphere on the Earth's surface.

For today, space geodesy is provided by the following basic methods of observations:
- Global Navigation Satellite Systems (GNSS);
- Satellite and Lunar Laser Ranging (SLR, LLR);
- Very Long Baseline Interferometry (VLBI);
- Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS);
- satellite altimetry;
- satellite gravimetry and others.

Each of these methods solves a certain range of tasks. Based on the analysis of works [1-3] we can generalize the possibilities of each of the chosen methods. Table 1 gives a general idea of the sensitivity of each of them to determine geodetic parameters.

This table shows: "+" are the parameters whose determination is potentially possible by the indicated observation method; "++" are the parameters regularly obtained from the processing of the corresponding method in the analysis centers; "+++" are the parameters for which specified method is the main one.

A revolutionary role in the development of geodesy, in particular existing methods for determining the location of geographic objects, was the emergence of global satellite-based navigation systems of coordinate-time implementation, such as GPS (USA) and GLONASS (Russia), which subsequently got the generic name of GNSS. With their help, it was possible to determine the coordinates of the observation points, including their altitude component, with high accuracy and efficiency.

GNSS is a complex electronic-technical system consisting of a set of ground and space equipment and is intended for positioning, determining the exact time and parameters of motion (velocity and direction) for ground, water and air objects [5]. Using GNSS to solve geodetic problems is the most widespread and massive method in geodesy.

Global navigation satellite systems include GPS, GLONASS, Galileo and Compass.
GNSS principle is to measure the distance from the antenna on the object of observation (whose coordinates are to be obtained) to satellites whose positions are known with high accuracy in the current time interval. On figure 1 it is a schematic shown the principle of this method.

Taking into account all corrections required for observations at station A to satellite S, general equation of this method will have the following form:

\[ \Delta \tau_S^{\text{GNSS}} = \left[ r_S^I - R e_{e,R} \right] + c \cdot \delta \tau_R - c \cdot \delta \tau_R + \delta \tau_{\text{trop},R} + \delta \tau_{\text{ion},R} + \delta \tau_{\text{phas},R} + \delta \tau_{\text{rel},R} + \delta \tau_{\text{mult},R} + \varepsilon_R^S \]  \hspace{1cm} (1)

where \( \Delta \tau_S^{\text{GNSS}} \) is a phase or code observation satellite S – station R;
\( c \) is a velocity of light;
\( r_S^I \) is a 3-dimensional position satellite S in an inertial reference frame (orbit parameters, coefficients of gravitational field);
\( R e_{e,R} \) is a 3-dimensional position of station R in a terrestrial reference frame (coordinates of the station);
\( R \) is a rotation matrix of Earth orientation (nutation, UT1, polar motion);
\( \delta \tau_R \) is a clock error on the station R (clock parameters on the station);
\( \delta \tau_{\text{trop},R} \) is a correction for tropospheric delay (tropospheric parameters);
\( \delta \tau_{\text{ion},R} \) is a correction for ionospheric delay (ionospheric parameters);
\( \delta \tau_{\text{rel},R} \) is a correction for relativistic effects;
\( \delta \tau_{\text{phas},R} \) is a the displacement error of the phase center of the satellite and receiver;
\( \delta \tau_{\text{mult},R} \) is a influence of ray multiplicity;
\( \varepsilon_R^S \) is a measurement error.

Taking into account all parts in the GNSS observation equation allows obtaining high-precision geodetic parameters.

SLR observations consist in measuring the length of time during which the ultrashort laser pulse travels from the ground station to the satellite and in the opposite direction [6]. Figure 2 shows schematically the principle of the SLR system.

The main observed value in the SLR is the measured time difference of the laser pulse from station R to satellite S and back. The multiplication of this time
interval with the speed of light gives a two-way distance between the station and the satellite. However, it is necessary to take into account corrections due to various influences on this method. As a result, the general equation of this method will look like this:

$$\frac{1}{2}c\Delta t^S_{S,SLR} = \left| r^S_t - R_{e,R} \right| + \delta_{tp,R}^S + \delta_{rel,R}^S + \delta_{bias,R}^S + \delta_{CoM,R}^S + \epsilon^S_R$$

(2)

$\Delta t^S_{S,SLR}$ is a time of the impulse towards station R - satellite S - station R$^S$;

c is a velocity of light;

$R_{e,R}$ is a 3-dimensional position of station R in a terrestrial reference frame (coordinates of the station);

$R$ is a rotation matrix of Earth orientation (nutation, UT1, polar motion);

$\delta_{tp,R}^S$ is a correction for tropospheric delay (tropospheric parameters);

$\delta_{rel,R}^S$ – is a correction for relativistic effects;

$\delta_{bias,R}^S$ is a systematic error of laser distance meter;

$\delta_{CoM,R}^S$ – correction for the incompatibility of the center of the masses of the satellite and the reflector;

$\epsilon^S_R$ – is a measurement error.

Having accumulated a sufficient number of differences between the observed and calculated topocentric distances, we can solve the problem of constructing internally coordinated parameters (system of orbit elements of the artificial satellites, orientation parameters of the Earth, coordinates of station positions, etc.) with the help of differential correction method.

From equations (1) and (2) we see that satellite observations can be used for studying of gravitational field of the Earth from analysis of changes in the position of geocentre. All equations contain coordinates of stations and orientation parameters of the Earth, therefore both methods can be used for the realization of the Earth’s coordinate system and obtaining a series of rotation parameters of the Earth.

Consolidated information about parameters that are evaluated by the GNSS and SLR methods is shown in Table 2.

The possibility of creating a common solution based on the data from various methods of space geodesy is due to the presence of parameters common to each of the methods of observations. There are three cases [7]:

1) identical parameters (for example, rotation parameters of the Earth, station velocities);

2) not identical parameters (for example, stations position on collocation platforms). In this case, for purpose of association, it is necessary to use geodetic reference data for stations coordinates communication of different observation methods at points of collocation;

3) unique parameters specific to one type of observations (for example, quasar position for VLBI, or constant corrections for SLR).

### Table 2

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<tr>
<th>Parameters derived from the processing of GNSS and SLR observations</th>
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<td><strong>Common parameters</strong></td>
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<td><strong>Global parameters</strong></td>
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<td>Nutation parameters</td>
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<td>Polar motion</td>
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<td>World time</td>
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<td>Length of day</td>
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<td>Location of geocenter</td>
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<td>Gravitational field</td>
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<td><strong>Local parameters</strong></td>
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<td>Station coordinates</td>
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<td>Tropospheric parameters</td>
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<td>Ionospheric parameters</td>
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<td>Antenna parameters</td>
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<td>Relativistic effect</td>
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<td><strong>Independent parameters</strong></td>
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<td>Clock parameters</td>
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<td>Satellite orbits</td>
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Delays in the troposphere are different for different methods due to the fact that observations are carried out in different ranges [3]. From the basic technologies of space geodesy, the SLR is least exposed to variations in the beam in the air channel. Compared to the microwave frequencies used in GNSS techniques, SLR optical frequencies are relatively insensitive to two of the most dynamic components of delayed passage through the atmosphere, ionosphere and water vapor distribution. Thus, the so-called “dry component” of the atmosphere is a major contribution to the error of passing the SLR signal [8].

As a result of mathematical manipulations over equations (1) and (2), we can express the signal delay through the troposphere as follows:

\[
\delta_{trp,R} = \frac{1}{2} (c \Delta \tau_R^{S,GNSS} + \frac{1}{2} c \Delta \tau_R^{S,SLR} - \\
- \delta_i^{S,GNSS} - R_{e,R} + c \delta_i^{S,GNSS} - c \delta_i^{S,SLR} - \\
- \delta_i^{S,GNSS} - \delta_i^{S,ion,R} - \delta_i^{S,phas,R} - \delta_i^{S,mul,SLR} - \delta_i^{S,GNSS} - \\
- \delta_i^{S,SLR} - R_{e,R} - \delta_i^{S,phas,R} - \delta_i^{S,SLR} - \\
- \delta_i^{S,rel,R} - \delta_i^{S,SLR} )
\]  

(3)

Conclusions

Advantages of modern methods of space geodesy are disclosed in full, subject to obtaining common solutions based on the results of simultaneous observations.

This article presents a general description of the GNSS and SLR methods of space geodesy. Based on the equations of these methods, was proposed the methodic for combining GNSS and SLR for determining tropospheric parameters. Long-term experience in the field of observation analysis and combination allows to get accurate geodetic products. The proposed method should be based on the use of geodetic data for modeling tropospheric parameters, without loss of accuracy and realization close to reality.

References


Method of determining the tropospheric parameters from compatible data of SLR and GNSS observations

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This paper presents the general characteristics of the two methods of space geodesy (GNSS and SLR), as well as analyzes the principles of creating common solutions in order to determine tropospheric parameters.