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HETEROGENUOUS STRUCTURES' FRACTAL ANALYSIS WITH THE VARIOGRAM METHOD APPLICATION

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Problem Stating

Recently instead of standard mathematical methods fractal geometry methods are used. It helps to give a quantitative description of heterogeneous objects and structures [1; 2; 3; 4]. Fractal geometry methods are turned to be effective in terms of the analysis of self-organization phenomena in dispersive systems.

Major preconditions, which the fractal geometry was based on, were observed at the beginning-middle of the previous century, and were combined only at the end of 70th by B.Mandelbrot. Fractal dimension, which is known as a quantitative measure of the object heterogeneity or self-similarity, became the main issue. It can be shown that for typical Euclidean figures, e.g. lines, spaces and distorted even surfaces, dimension is equal to 1, 2, 3 respectively. For the objects with the sharp heterogeneous structure dimension will not be full. In such cases it is widely accepted that those objects have fractal dimension.

Earth surface objects form and their spatial distribution are as much irregular that it is difficult to get satisfactory description with the help of geometrical methods.

Conformation, land-use structures are complex and shapeless from the point of view of Euclidean geometry indeed. Nevertheless they have high orderliness level and therefore can be characterized as stipulated chaotic structures. B. Mandelbrot outlined all the fractal dimensions of different relief type.

In the following papers [5] the fractal nature of different geographical structures is characterized. Ground cover structure also can be specified by the fractal dimension [1]. Though in the national cartographic-geographical literature there are only few publications in which the above-mentioned issue is outlined. These researches are essential for the Ukrainian local conditions.

Analysis of the latest research

Unconventional geometry, which is applied in fractal analysis, helps to reveal new current data about the object of the research, and gives a potential to complete the mathematical model of the object and to ensure relatively common description of the complex and irregular structure. These issues are fundamental and timely.

In the following papers [6;7] the theoretical substantiation of the fractal geometry issues in terms of new mathematical apparatus of the wavelet transformations is considered. Such approach is long-range and more detailed researches are needed. The publication [3], in which the algorithm of texture identification specialties with the Reni's fractal dimensions specters application is characterized, generates interest. Empirical support of the above-mentioned methodology is based on the analysis of the fractographic images. The separate researches are needed to introduce new opportunities of application in the digital photogrammetry.

The fractal geometry principles application in thematic mapping with the use of digital aerial images are considered in the following papers [3;9]. Fractal patterns of landforms are revealed in this work [9]. In the articles of Vasiliev L.N., Tyuflin A.S. sufficiently thorough theoretical and experimental results of the fractal analysis of digital images spatial geosystem's structures are specified.

Paper Outline

1. Variogram method

One of the frequently used methods of determining the fractal dimensions of digital images is variogram method [10;11]. This method is based on the assumptions that the digital image is a random changes process in the intensity of some pixels, and variograms are correct features of such process. Taking into consideration the importance of the fractal image analysis, variogram is can be substantiated as a characteristic of random function with stationary increments.

Considering the increments of the intensity image function $f(x) - f(x+h)$ instead of the function itself $f(x)$ the assumption of stationary increments is introduced, that is the mathematical expectation E is equal to zero. Dispersion Var is a function 2γ in terms of the distance between pixels h. The mathematical description of these assumptions is the following (1):

$$\begin{cases} E\left[f(x) - f(x+h)\right] = 0 \\ Var\left[f(x) - f(x+h)\right] = 2\gamma(h) \end{cases}. \qquad (1)$$

In the theory of random processes the function $2\gamma(h)$ is called variogram.

It is important to know the function of similarities or differences between the intensity values in pixels, appropriate to the digital image. This function can be called a spatial similarity function or spatial correlation function. The easiest way to compare two values $f(x)$ and $f(x+h)$ in pixels x and $x+h$, which have the distance h between each other. .

From the practical point of view it is more appropriate to handle values that do not depend on differences signs:

$$2\gamma(\vec{h}) = E\left[f(x) - f(x+\vec{h})\right]^2. \qquad (2)$$

The function $2\gamma(\vec{h})$ is a variogram. It is a vector argument function, in other words, it depends on the distance and direction. Variogram shows how the average intensity can vary, depending on the distance in a given direction. It should be noted that by the digital images analysis, the line of the image can be chosen. Suppose that in the line there is $N(h)$ pairs of intensity values which are at the distance in h pixels from each other, in that case variogram can be estimated by using the following expression:

$$2\gamma(h)=\frac{1}{N(h)}\sum_{i=1}^{N(h)}\left[f(x)-f(x+h)\right]^2. \quad (3)$$

Using the output intensity values of pixels in a line, the empirical variogram can be constructed and the theoretical model (mathematical function of approximation) can be chosen. Not all functions are suitable for describing the empirical variogram. The function which is set up by the following mathematical expression is widely used (4):

$$\gamma(h)=\begin{cases} C\times(1.5\frac{h}{a}-0.5(\frac{h}{a})^3)+C_0 & \text{при } 0<h\le a; \\ C+C_0 & \text{при } h>a; \\ 0 & \text{при } h=0. \end{cases} \quad (4)$$

That function (model) is called spherical.

The parameters of this model are the zone of influence, noise voltage dispersion and variogram threshold. The zone of influence (a) – distance between pixels, beyond which variogram acquires the character, reflecting the lack of correlation between the values of intensity. Beyond this zone the regularity in the intensities influence in pixels disappears. It is natural to characterize the zone of influence in a given direction (in terms of the order of images) value h from which the variogram reaches a certain threshold.

$C+C_0$ – variogram threshold, total variance of the image intensity, which is the limit variogram value. The variogram threshold consists of two constituent parts, one of which characterizes the random component dispersion, the second – regularity. The above-mentioned total variance can be regarded as the random component dispersion, imposed on the spatial variable dispersion. Thus, the total intensities variance in the line of digital images is the sum of the two dispersions.

In many cases variogram is not equal to zero when the distance h is equal to zero. This phenomenon may be the consequence of a strong change in the intensity values in pixels. These values are located at short distances from each other (this is connected with the presence of noise and local obstacles). Of course, this effect is associated with the random component. The parameter that characterizes the phenomenon of variogram C_0 is called the dispersion of noise. Thus, C_0 is a part of the total variance of image intensity due to the presence of noise. Then C - a part of the total variance due to the differences in intensity image levels of the object and background, as well as uneven intensity and same background and object. Fig. 1 shows the theoretical model parameters of the spherical form variogram.

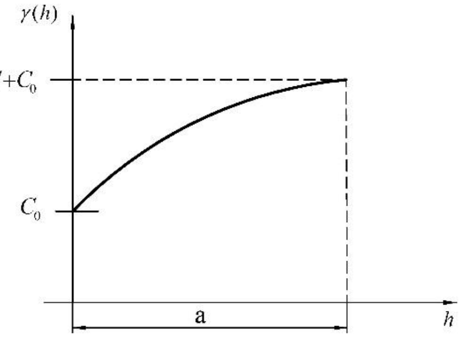

Fig. 1. Parameters of the theoretical spherical variogram : a – zone of the influence; C_0 – noise dispersion; $C+C_0$ – variogram threshold

2. Method of filtration

For getting the optimal variogram parameters, the previous pixel by pixel filtering of a digital image is required. For this the moving average method is used [11].

Considering the line of digital image under the guise which is equal to the zone of influence, it is needed to find a set of weighting coefficients a_i when $i=1,...k,n$ due to which the weighted average value $f(\tilde{x})$ is the best estimation of the intensity of the central pixel under the mask of:

$$f(\tilde{x})=\sum_{i=1}^{n}a_i f(x_i), \quad (5)$$

Where a_i – weighted coefficients; $f(\tilde{x})$ – new, filtered intensity value in the central pixel under the mask; $f(x_i)$ – intensities values in pixels under the mask.

Weighted coefficients a_i are identified with the help of the least square method. For doing that, the partial derivatives, which are equal to zero, are considered. Another way to determine the evaluation procedure quality is to find the variance of errors that occur.

It is known that the estimation error variance is given by the following formula:

$$\sigma_e^2=\sigma_0^2-2\sum_i a_i\sigma_{0i}+\sum_i\sum_j a_i a_j\sigma_{ij}, \quad (6)$$

where σ_e^2 – total intensity variance, the estimation of which by the variogram modeling is the threshold; σ_{0i} – intensity values covariance in the central pixel under the mask and in i – pixel; σ_{ij} – values covariance in i and j pixels; a_i, a_j – weighted coefficients, which determine the degree of each pixel influence under the guise of the intensity assessment in the central pixel. Covariance values, that are included in the expression (6), are determined on the basis of variogram as the difference $(C+C_0)$-$\gamma(h)$, where $C+C_0$ – variogram threshold, and $\gamma(h)$ – variogram magnitude of a given value h the following coefficients $a_i(a_j)$ can be used to minimize σ_e^2, to find the weighted average of the smallest variance errors. The required estimation should not have systematic errors, it must be performed under the unbiasedness condition $\sum_i a_i = 1$.

Thus, taking into account the additional conditions the function F should be minimized, not the function σ_e^2:

$$F = \sigma_e^2 + 2\mu\left(\sum_i a_i - 1\right), \qquad (7)$$

where μ – Lagrange multiplier.

The condition of a function F minimum is the vanishing of all the partial derivatives a_i i μ:

$$\frac{\partial F}{\partial a_i} = -2\sigma_{0i} + 2\sum_j a_j \sigma_{ij} + 2\mu = 0, \quad i = 1,...,n;$$
$$\frac{\partial F}{\partial \mu} = \left(\sum_i a_i - 1\right) = 0. \qquad (8)$$

It is a system $n+1$ of linear equations with $n+1$ and unknowns a_i and μ.

In traditional form of writing system of equations is:

$$\begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} & 1 \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} & 1 \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_n \\ \mu \end{bmatrix} \cdot \begin{bmatrix} \gamma_{01} \\ \gamma_{02} \\ \gamma_{0n} \\ 1 \end{bmatrix}, \qquad (9)$$

where γ_{0i} – thjeoretical variogram value; γ_{ij} – empirical variogram value; a_i –coefficients; μ – Lagrange multiplier.

3. Determination of the digital images fractal dimension by the variogram method

Practical realization of the variogram method is accomplished in the following way. Serial profile of the intensities changes is specified by the defined length windows [13].

The average value of the sum of squares and intensity values differences between the points of the window width are calculated. Fig. 2 shows the calculations scheme for the two windows with the height of h (fig.2, a), and $2h$ (fig. 2, б).

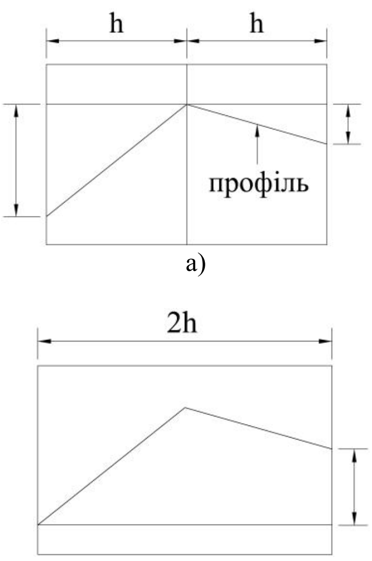

Fig. 2. The scheme of intensity difference calculation

According to this scheme the empirical variogram function is calculated $\gamma(h)$:

$$\gamma(h) = \frac{1}{2N}\sum_{i=1}^{N}\left[g(x_i) - g(x_i + h)\right]^2, \qquad (10)$$

where N – the total number of the profile points pairs with the intensity $g(x_i)$ and $g(x_i + h)$. These points are located at the borders of the windows with the width h.

The function $\gamma(h)$ is calculated for some values h, then the dependency graph is drawn $\log[\gamma(h)]$ from $\log[h]$. The slope of the line β, that approximate this graph, the fractal profile dimension is calculated in the following way $D_f = 2 - \beta/2$.

We tried to illustrate the theoretical calculations by the concrete example

According to [13] matrix equation is:

$$\begin{bmatrix} 0 & 1{,}89 & 1{,}89 & 1{,}89 & 1{,}89 & 1 \\ 1{,}89 & 0 & 2 & 2 & 2 & 1 \\ 1{,}89 & 2 & 0 & 2 & 2 & 1 \\ 1{,}89 & 2 & 2 & 0 & 2 & 1 \\ 1{,}89 & 2 & 2 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ \mu \end{bmatrix} = \begin{bmatrix} 0{,}88 \\ 1{,}86 \\ 1{,}86 \\ 1{,}86 \\ 1{,}86 \\ 1 \end{bmatrix}. \qquad (11)$$

From the solution (11)we get:

$a_1 = 0{,}60$;

$a_2 = a_3 = a_4 = a_5 = 0{,}10$, $\overset{5}{\underset{i=1}{Z}} a_i = 1$.

Fractal dimension :

$D_f = 2 - \beta/2 = 2 - 0{,}628/2 = 1{,}686$.

To describe heterogeneous structures in most cases two approaches - statistical and textural - are used The main difficulty in studying the textures properties is that it is difficult to develop a universal way of textures recognition. In other words, practically any type of texture can be customized to the method of recognition that with the appropriate setting will provide sufficiently reliable results, and with the other type of texture it may not work at all. There is one reson to describe the complex texture, when the Euclidean geometry is commonly used. Euclidean dimensions can characterize the symmetrical textures, which are not very common in photographic images, such as of cultivated areas etc. Therefore, to describe such images the fractal geometry, which can be characterized by the fractal dimensions spectrum (multifractality), should be used.

Multifractal spectrum is a set of fractal dimensions D_g, which in its turn can be represented by some nonlinear function $\tau(g)$

The values D_g are not fractal dimensions in conventional sense, therefore the so-called multifractal spectrum $f(\alpha)$ function is used. It is obtained by the Legendre function $\tau(g)$ converting:

$$f(\alpha(g)) = \alpha(g) \cdot g - \tau(g). \qquad (12)$$

The algorithms of the components $f(\alpha(g))$ calculation are outlined in the following paper [3].

As a result of these calculations we get the set of values D_g, on the basis of which the comparative texture images identification procedure. The varieties of measures are used. It may be different types of Euclidean distances, Hamming distance, Chebyshev distance etc. For example, Chebyshev distance corresponds to the value of the difference module between the values of relevant textural properties of certain object (image) areas:

$$\alpha = \max_{g}\left|D_g - D_g^{'}\right|. \qquad (13)$$

4. **The distinction of the borders (edges) in heterogeneous images elements**

Another important objective of the heterogeneous structures image analysis is the allocation of particular segments limits or image areas. The simplest and mostly-used method is the so-called marginal discrimination method. [12]. However, the matrix of brightness G is converted into binary matrix B in which non-zero elements correspond to the value of the output matrix which is bigger than fixed threshold $g_{поріг}$. Let us supposed the method of marginal discrimination in the following way:

$$G = \begin{pmatrix} g_{11} & \cdots & g_{1N} \\ \vdots & \ddots & \vdots \\ g_{M1} & \cdots & g_{MN} \end{pmatrix} \Rightarrow B = \begin{pmatrix} b_{11} & \cdots & b_{1N} \\ \vdots & \ddots & \vdots \\ b_{M1} & \cdots & b_{MN} \end{pmatrix}, \qquad (14)$$

$$b_{ij} = \begin{cases} 1, \ якщо \ g_{ij} > g_{поріг} \\ 0, \ якщо \ g_{ij} < g_{поріг} \end{cases}, \qquad (15)$$

where M and N – matrix pixel size of the image partition, $g_{поріг}$ – threshold constant.

Software implementation of the given algorithm was accomplished in AP «Stiman». To find the areas of interest in half-toned PEM-images in AP «Stiman». The method of lands allocation was taken as a basis. The edges (borders) are characterized by the fact that along them the sharp function change in brightness or its derivatives on spatial variables is stipulated. This behavior of the brightness function is caused by various physical reasons. This can be either element structure border, or saltatory variation, or one of the other depicted objects In most boundary structures of these elements there are borders, which allows to use filtering by the so-called Laplacian of gaussian (LoG):

$$G = \begin{pmatrix} g_{11} & \cdots & g_{N1} \\ \vdots & \ddots & \vdots \\ g_{1M} & \cdots & g_{NM} \end{pmatrix} \Rightarrow C = \begin{pmatrix} c_{11} & \cdots & c_{N1} \\ \vdots & \ddots & \vdots \\ c_{1M} & \cdots & c_{NM} \end{pmatrix}, \qquad (16)$$

$$c(x,y) = \sum_{\alpha=-\infty}^{+\infty}\sum_{\beta=-\infty}^{+\infty} g(\alpha,\beta) \times LoG(x-\alpha, y-\beta), \qquad (17)$$

$$g(x,y) = \begin{cases} g_{x-1,y-1}, \ якщо \ 1 \le x \le N, 1 \le y \le M \\ 0, \ для \ решти \ (x,y) \end{cases}, \qquad (18)$$

where $c(x,y)$ – two-dimentional discrete convolution value, $g(x,y)$ –Laplacian Gaussian function of brightness $LoG(x,y)$.

Let us suppose that two structural areas A and $Б$ are under the estimation and these data have a common border N, area A marked by value N_A and area $Б$ – $N_Б$. If under the border crossing in the direction from the area A into the area $Б$ the symbol of the second value changes from positive into negative, then $N_A < N_Б$, if the symbol changes from negative into positive one, then $N_A > N_Б$.

Summary and further research prospects

This paper proposes an original method of identification for automatic detection of various types of heterogeneous structures, such as farmlands. It is proposed to apply the principles of images multifractality and to use Chebyshev distance minimum as the criterion.

The definition of the digital images fractal dimension, using the method variogram, is substantiated. This approach is optimal for the fractal metrics estimation (Hausdorff-Besicovitch dimensions, Rainier dimensions etc). The algorithms of the borders, territories, regions and certain segments of digital images of heterogeneous structures.recognition are proposed.

The issued considered in the article are innovative and require extensive further researches. In particular, it is desirable to explore the use of different variogram types, the accuracy of the parameters determination and their influence on the accuracy of the fractal dimensions distinction. The criteria of heterogeneous structures identification should be optimized. Diverse experimental researches should be done.

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Heterogenuous structures' fractal analysis with the variogram method application

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The paper explores the issues of heterogenuous structures' fractal analysis. This method is based on the intensity of the aerial digital images processing. It is proposed to define Hausdorff–Besicovitch fractal dimension using a variogram method. Analyzing heterogeneous image structures, the issue of the boundaries (edges) determination of certain image areas (segments) is of great importance. In this article the algorithm which deals with this problem is considered.