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ARGUMENTATION OF ACCURACY OF LINEAR SEGMENTS BY MEASURING OF LINES TO THEIR ENDS BY ELECTRONIC TACHYMETER

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Formulation of the problem

Using the power of modern tachymeters, previously there have been conducted researches of an opportunity to determine linear intervals by measuring distances to their ends and angles using reflective films [2].

Analysis of the results has shown the dependence of the root-mean-square error of the determined linear segment length on the accuracy of the measured angle and distance from electronic tachymeter to the ends of this segment. The proposed method satisfies the required accuracy of calibration of staffs allocated for III - IV leveling classes. However, in this particular case the method is considered such a definition, in connection with which there is an opportunity to explore other options for solving this problem.

Analysis of recent researches and publications

Determination of short distances with high accuracy [3], requires special devices and equipment. Calculation of meter length values with high accuracy requires standard [3], which must be checked in Kharkiv Meteorological Institute. In our case calibration of target rods has been made by controlling meter rod or invar rod at the comparator MK-1. Special devices are required to measure phase frequency. Works [1, 4] describe the method that allows calculating linear values with high accuracy using electronic tachymeter.

Statement of facts of material

The study, presented in [4], submits a reasoned confirmation of equality of two parties in determining the error of arm calculation.

Thereby the problem will be set in general:

according to determined (measured) sides a_1, a_2 and the angle α between them by the law of cosines the side can be found $l=\sqrt{a_1^2+a_2^2-2a_1a_2\cos\alpha}$,

root-mean-square error l is calculated by the formula:

$$m_l=\frac{1}{l}\sqrt{\begin{matrix}(a_1-a_2\cos\alpha)^2 {m_{a_1}}^2+(a_2-a_1\cos\alpha)^2 {m_{a_2}}^2\\ +a_1^2a_2^2\sin^2\alpha\, m_\alpha^2\end{matrix}}, \qquad (1)$$

where

$$m_l l=\sqrt{\begin{matrix}(a_1-a_2\cos\alpha)^2 {m_{a_1}}^2+(a_2-a_1\cos\alpha)^2 {m_{a_2}}^2\\ +a_1^2a_2^2\sin^2\alpha\, m_\alpha^2\end{matrix}}. \qquad (2)$$

Value m_l depends on the quantities a_1, a_2 and α, so, therefore, you can set the minimality condition (except for the obvious minimum $m_\alpha=0$). The existence of extremum function of three variables is as follows:

$$\begin{cases}\dfrac{\partial(m_l)}{\partial a_1}=\dfrac{1}{m_l l^2}\begin{bmatrix}(a_1-a_2\cos\alpha)m_{a_1}{}^2-\cos\alpha(a_2-a_1\cos\alpha)m_{a_2}{}^2\\ +a_1a_2^2\sin^2\alpha\, m_\alpha^2-m_l{}^2(a_1-a_2\cos\alpha)\end{bmatrix}=0\\ \dfrac{\partial(m_l)}{\partial a_2}=\dfrac{1}{m_l l^2}\begin{bmatrix}-\cos\alpha(a_1-a_2\cos\alpha)m_{a_1}{}^2+(a_2-a_1\cos\alpha)m_{a_2}{}^2\\ +a_1^2a_2\sin^2\alpha\, m_\alpha^2-m_l{}^2(a_2-a_1\cos\alpha)\end{bmatrix}=0\\ \dfrac{\partial(m_l)}{\partial\alpha}=\dfrac{\sin\alpha}{m_l l^2}\begin{bmatrix}(a_1-a_2\cos\alpha)a_2m_{a_1}{}^2+(a_2-a_1\cos\alpha)a_1m_{a_2}{}^2\\ +a_1^2a_2^2\cos\alpha\, m_\alpha^2-a_1a_2\, m_l{}^2\end{bmatrix}=0\end{cases} \qquad (3)$$

It has been established that the non-linear system of equations (3) has no solutions. It has also been confirmed by numerical experiments.

Additional conditions for its existence have been brought, namely, to solve the problem of conditional extremum so that the Lagrange function will be introduced:

$$\begin{gathered}m_l(a_1,a_2,\alpha,\lambda)=-\lambda\left(l-\sqrt{a_1^2+a_2^2-2a_1a_2\cos\alpha}\right)\\ +\frac{1}{l}\sqrt{(a_1-a_2\cos\alpha)^2 {m_{a_1}}^2+(a_2-a_1\cos\alpha)^2 {m_{a_2}}^2+a_1^2a_2^2\sin^2\alpha\, m_\alpha^2}.\end{gathered} \qquad (4)$$

In this case the system of equations (3) takes the form:

$$
\begin{cases}
\dfrac{\partial(m_l)}{\partial a_1}=\dfrac{1}{m_l l^2}\begin{bmatrix}(a_1-a_2\cos\alpha)\cdot {m_{a_1}}^2-\cos\alpha(a_2-a_1\cos\alpha)\cdot {m_{a_2}}^2\\ +a_1a_2^2\sin^2\alpha\cdot m_\alpha^2-{m_l}^2\cdot(a_1-a_2\cos\alpha)\end{bmatrix}\\
-\dfrac{\lambda}{l}(a_1-a_2\cos\alpha)=0\\
\dfrac{\partial(m_l)}{\partial a_2}=\dfrac{1}{m_l l^2}\begin{bmatrix}-\cos\alpha(a_1-a_2\cos\alpha)\cdot {m_{a_1}}^2+(a_2-a_1\cos\alpha)\cdot {m_{a_2}}^2\\ +a_1^2a_2\sin^2\alpha\cdot m_\alpha^2-{m_l}^2\cdot(a_2-a_1\cos\alpha)\end{bmatrix}\\
-\dfrac{\lambda}{l}(a_2-a_1\cos\alpha)=0\\
\dfrac{\partial(m_l)}{\partial \alpha}=\dfrac{\sin\alpha}{m_l l^2}\begin{bmatrix}(a_1-a_2\cos\alpha)a_2\cdot {m_{a_1}}^2+(a_2-a_1\cos\alpha)a_1\cdot {m_{a_2}}^2\\ +a_1^2a_2^2\cos\alpha\cdot m_\alpha^2-{m_l}^2\cdot a_1a_2\end{bmatrix}\\
-\dfrac{\lambda}{l}a_1a_2=0\\
l=\sqrt{a_1^2+a_2^2-2a_1a_2\cos\alpha}
\end{cases}
\quad (5)
$$

Having completed the algebraic conversion, the system of equations (5) can be written:

$$
\begin{cases}
\dfrac{a_1a_2}{m_l l^2}\begin{bmatrix}(a_1-a_2\cos\alpha){m_{a_1}}^2-\cos\alpha(a_2-a_1\cos\alpha){m_{a_2}}^2\\ +a_1a_2^2\sin^2\alpha\cdot m_\alpha^2-(a_1-a_2\cos\alpha)\\ \left((a_1-a_2\cos\alpha)a_2{m_{a_1}}^2+(a_2-a_1\cos\alpha)a_1{m_{a_2}}^2+a_1^2a_2^2\cos\alpha\cdot m_\alpha^2\right)\end{bmatrix}=0\\
\dfrac{a_1a_2}{m_l l^2}\begin{bmatrix}-\cos\alpha(a_1-a_2\cos\alpha){m_{a_1}}^2+(a_2-a_1\cos\alpha){m_{a_2}}^2\\ +a_1^2a_2\sin^2\alpha\cdot m_\alpha^2-(a_2-a_1\cos\alpha)(a_1-a_2\cos\alpha)a_2{m_{a_1}}^2\\ +(a_2-a_1\cos\alpha)a_1{m_{a_2}}^2+a_1^2a_2^2\cos\alpha\cdot m_\alpha^2\end{bmatrix}=0\\
\cos\alpha=\dfrac{a_1^2+a_2^2-l^2}{2a_1a_2}
\end{cases}
\quad (6)
$$

From the first two equations of the system (6) and using the third equation we exclude expressions dependent on α .

We introduce the denotation $d=l^2+a_2^2$, $r=a_2^2-l^2$, then, we get:

$$\cos\alpha=\frac{a_1^2+r}{2a_1a_2},\quad \sin^2\alpha=\frac{-a_1^4+2a_1^2d-r^2}{4a_1^2a_2^2},$$

$$a_1-a_2\cos\alpha=\frac{a_1^2-r}{2a_1},\quad a_2-a_1\cos\alpha=\frac{d-a_1^2}{2a_1},$$

$$\cos\alpha\left(a_1-a_2\cos\alpha\right)=\frac{a_1^4-r^2}{4a_1^2a_2},$$

$$\cos\alpha\left(a_2-a_1\cos\alpha\right)=\frac{-a_1^4+a_1^2\left(d-r\right)+rd}{4a_1a_2^2}.$$

place

$$c_0=\frac{1}{2}\left(-m_{a_2}^2+a_2^2m_\alpha^2\right),c_1=0,$$

$$c_2=\frac{1}{2}\left(a_2^2m_{a_1}^2+dm_{a_2}^2+ra_2^2m_\alpha^2\right),c_3=0,$$

$$c_4=-\frac{1}{2}ra_2^2m_{a_1}^2,$$

and also

$$b_0=\frac{1}{4}\left(m_{a_2}^2-a_2^2m_\alpha^2\right),b_1=0,$$

$$b_2=\frac{1}{4}\left(2a_2^2m_{a_1}^2-\left(d-r\right)m_{a_2}^2+2da_2^2m_\alpha^2\right),$$

$$b_3=0,b_4=\frac{1}{4}\left(2ra_2^2m_{a_1}^2-rdm_{a_2}^2-r^2a_2^2m_\alpha^2\right),$$

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$$u_0=\frac{m_\alpha^2}{4},u_1=0,u_2=\frac{1}{4}\left(-m_{a_1}^2-2m_{a_2}^2+2dm_\alpha^2\right),$$

$$u_3=0,u_4=\frac{1}{4}\left(2dm_{a_2}^2-r^2m_\alpha^2\right),$$

$$u_5=0,u_6=\frac{r^2}{4}m_\alpha^2.$$

Then to solve the following system we introduce the denotation:

$$v_1=b_0-\frac{c_0}{2},\quad v_2=b_1-\frac{c_1}{2},\quad v_3=b_2-\frac{c_2}{2}+\frac{rc_0}{2},$$

$$v_4=b_3-\frac{c_3}{2}+\frac{rc_1}{2},\quad v_5=b_4-\frac{c_4}{2}+\frac{rc_2}{2},$$

$$v_6=\frac{rc_3}{2},\quad v_7=\frac{rc_4}{2};$$

$$w_1=a_2^2u_0+\frac{c_0}{2},\quad w_2=a_2^2u_1+\frac{c_1}{2},$$

$$w_3=a_2^2u_2+\frac{c_2}{2}-\frac{dc_0}{2},\quad w_4=a_2^2u_3+\frac{c_3}{2}-\frac{dc_1}{2},$$

$$w_5=a_2^2u_4+\frac{c_4}{2}-\frac{dc_2}{2},\quad w_6=a_2^2u_5-\frac{dc_3}{2},$$

$$w_7=a_2^2u_6-\frac{dc_4}{2}.$$

The system (6) is reduced to the system of two algebraic equations:

$$
\begin{aligned}
F\left(a_1,a_2\right)&=\sum_{i=1}^{7}a_1^{7-i}v_i\left(a_2\right)=0\\
G\left(a_1,a_2\right)&=\sum_{i=1}^{7}a_1^{7-i}w_i\left(a_2\right)=0,
\end{aligned}
\quad (7)
$$

which has common solution regarding a_1 when the determinant (7) (resolvent) [5], is composed of coefficients $v_i\left(a_2\right)$, $w_i\left(a_2\right)$ equals zero.

Solving it regarding variable a_2, and then after substituting into one of the equations of the system and variable a_1, we obtain results that coincide with the above in work [1].

$$
\det(a_2)=\begin{vmatrix}
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & 0 & 0 & 0 & 0 & 0\\
0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & 0 & 0 & 0 & 0\\
0 & 0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & 0 & 0 & 0\\
0 & 0 & 0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & 0 & 0\\
0 & 0 & 0 & 0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & 0\\
0 & 0 & 0 & 0 & 0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7\\
w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & 0 & 0 & 0 & 0 & 0\\
0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & 0 & 0 & 0 & 0\\
0 & 0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & 0 & 0 & 0\\
0 & 0 & 0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & 0 & 0\\
0 & 0 & 0 & 0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & 0\\
0 & 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7
\end{vmatrix}=0
\quad (8)
$$

Table 1

The value of the optimal values of the parties to determine the arm

№	$l(мм)$	$m_{a_1}(мм)$	$m_{a_2}(мм)$	$m_\alpha \times 1''$	$a_1(мм)$	$a_2(мм)$	$m_l(мм)$
1	0.1	2	2	5	2.412	2.412	0.083
2	1.0	2	2	5	7.638	7.638	0.262
3	3.0	2	2	5	13.223	13.223	0.452
4	0.1	1	1	2	2.7000	2.7000	0.037
5	1.0	1	1	2	8.540	8.540	0.118
6	3.0	1	1	2	14.791	14.791	0.202

Results

The research made it possible to obtain formulas for calculating the root-mean-square error of the linear interval in a general case. The numerical implementation of the algorithm has shown the existence of one extremum for the error in calculating of the length of a side according to the measurements.

The value of errors that was obtained by the mentioned method and in previous studies is virtually identical. Therefore, in future it is advisable to use the algorithm given above.

The optimal value of the measured sides and the angle between them is found in minimality of the error while calculating the linear interval and could be the basis for solving other specific engineering problems.

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Argumentation of accuracy of linear segments by measuring of lines to their ends by electronic tachymeter

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We have considered the general case of definition of arm length measured by the sides and the angle between them and established its calculation error.
The conditions under which it takes a minimum value in the case of an arbitrary allocation unit are defined. We have also determined the lack of existence of the extremum. For the correctness of solving the problem additional conditions are required (for example, determining fixation side). The algorithm enables to find optimal value of the sides when root-mean-square error becomes minimal. By numerical experiments it has been established the existence of unique solution that matches the case of fixing the alignment measuring tachymeter side (measuring sides are the same). For identical data the results of calculations by different methods coincide. The procedure described in the paper can be used to determine the length decimeter and meter intervals of target rods that are used in II, III and IV class leveling.