ON THE TRANSFORMATION OF THE SPATIAL COORDINATES WITH THE GIBBS VECTOR FORMALISM APPLICATION

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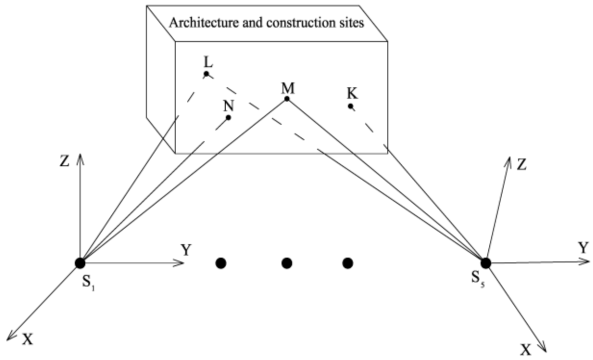
Key wods: Gibbs vectors, spatial orientation, coordinate system, resulting slopes.

Problem Stating

The necessity of spatial rectangular coordinates transformation from one system into another is happened very often in geodesy and photogrammetry [1,2]. Having made survey from different shooting points by means of electron tachymeter (pic.1), e.g., reconstructed buildings, all the shooting results are conducted in one common model.



Pic. 1 Before the coordinates transformation (1).

Speaking about photogrammetric survey there is always a necessity of coordinate system transformation. However, in photogrammetry this process is divided into 2 stages. The first stage – these surveys, stereopairs rectangular components are mutually oriented in one of the well-known methods aside from each other and receive separate models in theirs coordinate system [1], e.g., $S_1X_1Y_1Z_1$ and $S_5X_5Y_5Z_5$. There are no difficulties in that stage. The next stage is more complicated. These models are combined in common model in the integrated coordinate system, e.g. , in $S_1X_1Y_1Z_1$. For that well-known formulas of transition from one coordinate system into another may be used [2]:

$$R_1 = R_0 + A_5 R_5 t . \qquad (1)$$

In our case R_1 i R_5 - vectors that take up a position of the point M_1 of coordinate system model $S_1X_1Y_1Z_1$ and $S_5X_5Y_5Z_5$ respectively; R_0 - vector that sets a position of the beginning of coordinate systems $S_5X_5Y_5Z_5$ with regard to $S_1X_1Y_1Z_1$; A_5 - orthogonal matrix of one coordinate system turn concerning the other coordinate system; t - is a scale factor of the second model relative to the first one.

To take advantage of equation (1), it is necessary to know R_0 A_5, t, the meaning of which may be defined by the common points of two models. This well-known method of discovering these values demands the knowledge of approximate meanings of model orientation elements. We may gain them as a result of field geodesic measurement performing. However, this process is labor intensive and difficult, esp. for the angular attitude data getting (pic.1). Therefore, it is offered to dispense with the approximate elements meanings in model orientation. For that the beginning of coordinate system of the first and the second models $S_1X_1Y_1Z_1$ and $S_5X_5Y_5Z_5$ in any of the common points, e.g., in the point M_1 or in the centre of model points weight. Then the equation (1) is converted into:

$$R_{1M} = A_5 R_{5M} t , \qquad (2)$$

where R_{1M} and R_{5M} - vectors that define the position of the model point in coordinate systems $M_1X_1Y_1Z_1$ i $M_1X_5Y_5Z_5$ respectively. For the angular attitude of the converted coordinate systems mathematical Gibbs vectors formalism is used [3,4].

There is a distinctive feature of the Gibbs vector method – it is a possibility to define common orientation of two models at any angles (from 0º to 360º).

Connection of the following topic with the significant scientific and practical tasks.

Research data are connected with the research scientific work “Investigation of the modern state and the development of the rational land use of the erosive and degraded lands of Volyn vysochyna (hill) basis by means of GIS-technologies and SEM-microscopy” (№ state registration 0111 U 002146), they were conducted by Professors of the Department of geodesy, land exploitation and cadastre Lesya Ukrainka Eastern European University (2011-2012р.).

Analysis of the latest research and publications.

The topicality of this paper is confirmed in [9,10], in which the significance of this theme is emphasized.

Aim Stating

The object of this paper is to concretize Gibbs vectors mathematical formalism in reference to the tasks of spatial orientation of coordinate systems and to get the locution of the resulting slope in vector form.

Paper Outline

I. Description of the spatial orientation of coordinate systems by means of Gibbs vectors

Physically any turn represents turning in angle Ω in axial rotation, that characterizes by means of unit vector $\vec{C}$ with directive cosines (C_1, C_2, C_3) [2,5,6]. Transformation law i vector $\vec{g}_{(i)}$ into $\vec{r}_{(i)}$ looks as a following:

$$\vec{r}_{(i)} = \hat{B}\vec{g}_{(i)}, \qquad (3)$$

where $\hat{B}$ - access statement.

E.g., n electron microscopy of chips, as one of the effective method of $\hat{B}$ operator identification, Gibbs vector formalism may be used $\vec{G}$ [7]. Put down Gibbs vectors in traditional form. Gibbs vector, as is generally known [3], is set by the three components $\vec{G}_1, \vec{G}_2, \vec{G}_3$. Vector $\vec{G}$ components are connected with the theta displacement Ω and directive cosines (C_1, C_2, C_3) of unit vector $\vec{C}$, which is oriented lengthwise the axis of rotation. That connection looks like [2]: $\vec{G} = \vec{C} \cdot tg\frac{\Omega}{2}$. Mathematical vector transformation $\vec{g}_{(i)}$ into $\vec{r}_{(i)}$ is converted in the following way:

$$\vec{r}_{(i)} = \cos^2\frac{\Omega}{2}\left[\left(1-\left|G^2\right|\right)\vec{g}_{(i)} + 2\left(\vec{G}\cdot\vec{g}_{(i)}\right)\vec{G} + 2\vec{G}\times\vec{g}_{(i)}\right] \qquad (4)$$

Here vector and scalar product should be written by means of ordinary formulas, and vectors $\vec{r}_{(i)}$, $\vec{g}_{(i)}$, $\vec{G}$ - in unitary coordinate system. With the aim of calculation's simplification [5] of the next characteristics of Gibbs vector $\vec{G}$ (4) the following formula is used:

$$\vec{G}\times\left(\vec{r}_{(i)} + \vec{g}_{(i)}\right) = \left(\vec{r}_{(i)} - \vec{g}_{(i)}\right). \qquad (5)$$

Having used that characteristic the difference is minimized

$$F(G_1, G_2, G_3) = \sum_{i=1}^{N}\left[\vec{G}\times\left(\vec{r}_{(i)} + \vec{g}_{(i)}\right) - \left(\vec{r}_{(i)} - \vec{g}_{(i)}\right)\right]^2 \xrightarrow{G_1, G_2, G_3} \min \qquad (6)$$

In contrast to (4) relation (6) is linear to G_1, G_2, G_3.

After the transition from vector to coordinate form and simple transformations, we received the system of three equations with the three unknowns G_1, G_2, G_3, in the following way:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \qquad (7)$$

where

$$a_{11} = \sum_{i=1}^{N}\left[\left(\vec{r}_{i2} + g_{i2}\right)^2 + \left(\vec{r}_{i3} + g_{i3}\right)^2\right];$$

$$a_{22} = \sum_{i=1}^{N}\left[\left(\vec{r}_{i1} + g_{i1}\right)^2 + \left(\vec{r}_{i3} + g_{i3}\right)^2\right];$$

$$a_{33} = \sum_{i=1}^{N}\left[\left(\vec{r}_{i1} + g_{i1}\right)^2 + \left(\vec{r}_{i2} + g_{i2}\right)^2\right];$$

$$a_{lm} = -\sum_{i=1}^{N}\left(\vec{r}_{il} + g_{il}\right)\left(\vec{r}_{im} + g_{im}\right), l \neq m;$$

$$b_1 = 2\sum_{i=1}^{N}\left(g_{i2}\vec{r}_{i3} - g_{i3}\vec{r}_{i2}\right);$$

$$b_2 = 2\sum_{i=1}^{N}\left(g_{i3}\vec{r}_{i1} - g_{i1}\vec{r}_{i3}\right);$$

$$b_3 = 2\sum_{i=1}^{N}\left(g_{i1}\vec{r}_{i2} - g_{i2}\vec{r}_{i1}\right).$$

The right part contains only experimentally definable parameters of the fulcrum vectors in two coordinate systems $\vec{r}_{(i)}$ and $\vec{g}_{(i)}$. Measured quantities error is taken into consideration for the estimation of measurements stand precision

$$s = \frac{1}{N-1}\sum_{i=1}^{N}\left(\vec{r}_{(i)} - \vec{r}_{(i)}\right)^2, \qquad (8)$$

where $\vec{r}_{(i)}$ is calculated by means of the following formula (4).

II. Modification of resulting slope vector definition.

Resulting slope has fundamental meaning as for chips research and spatial structure of complicated dislocation configuration, when in the sample the selected slopes should be mentioned [7]. This question will be regarded in the next stating, having used $\vec{w}_0$ mark, as in the electron microscopy.

Let turn about axis characterizes by means of vector $\vec{w}_0$, the direction of which coincides with the rotation axis and module is equal to $tg\delta/2$, i.e. $|\vec{w}_0| = tg\delta/2$. In that case resulting turn vector which is get from the two successive turns with $\vec{w}_1$ and $\vec{w}_2$ vectors, is calculated by the following formula [8]:

$$\vec{w} = \frac{\vec{w}_1 + \vec{w}_2 - \vec{w}_1\times\vec{w}_2}{1 - \vec{w}_1\cdot\vec{w}_2}. \qquad (9)$$

Here $\vec{w}_1\times\vec{w}_2$ - vector product, $\vec{w}_1\cdot\vec{w}_2$ - scalar product. Let $|\vec{w}_1| = tg\varphi/2$; be $|\vec{w}_2| = tg\psi/2$. Then $|\vec{w}_1 + \vec{w}_2| = \sqrt{|\vec{w}_1|^2 + |\vec{w}_2|^2} = \sqrt{tg^2\varphi/2 + tg^2\psi/2}$

and with taking into consideration vector normalcy $\vec{w}_1$ and $\vec{w}_2$

$$ctg\varepsilon = \pm\frac{|\vec{w}_1 + \vec{w}_2|}{|\vec{w}_1 \times \vec{w}_2|} = \pm\frac{\sqrt{tg^2\varphi/2 + tg^2\psi/2}}{tg\varphi/2 + tg\psi/2}$$

Resulting angle δ, is the same as rotation axis turning $\vec{w}$ is equal to $\mathrm{tg}\delta/2 = |\vec{w}|$. Therefore

$$\vec{w}^2 = \vec{w}_1^{\,2} + \vec{w}_2^{\,2} + (\vec{w}_1 \times \vec{w}_2)^2 - 2\vec{w}_1(\vec{w}_1 \times \vec{w}_2) - 2\vec{w}_2(\vec{w}_1 \times \vec{w}_2) = \vec{w}_1^{\,2} + \vec{w}_2^{\,2} + (\vec{w}_1 \times \vec{w}_2)^2$$

; $\vec{w}_1(\vec{w}_1 \times \vec{w}_2)^2 = 0$. Finally

$$\mathrm{tg}\delta/2 = |\vec{w}| = \sqrt{tg^2\varphi/2 + tg^2\psi/2 + tg^2\varphi/2 \cdot tg^2\psi/2}\,. \tag{10}$$

Having known resulting slope, transformation operator B may be calculated by means of well-known formula [9].

$$B = \begin{bmatrix} (1-\sigma_1^2)\cos\delta + \sigma_1^2 & \sigma_1\sigma_2(1-\cos\delta) + \sigma_3\sin\delta & \sigma_1\sigma_3(1-\cos\delta) - \sigma_2\sin\delta \\ \sigma_1\sigma_2(1-\cos\delta) - \sigma_3\sin\delta & (1-\sigma_2^2)\cos\delta + \sigma_2^2 & \sigma_2\sigma_3(1-\cos\delta) + \sigma_1\sin\delta \\ \sigma_1\sigma_3(1-\cos\delta) + \sigma_2\sin\delta & \sigma_2\sigma_3(1-\cos\delta) - \sigma_1\sin\delta & (1-\sigma_3^2)\cos\delta + \sigma_3^2 \end{bmatrix}$$

,(11)

where $\sigma_1, \sigma_2, \sigma_3$ - directive cosines of rotation axis.

Summary

Verification of the correctness of expounded method is accomplished on mathematical models. That verification confirmed the effectiveness of that method in geodesy and photogrammetry by using modern computes.

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On the transformation of the spatial coordinates with the application of Gibbs vector formalism

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The paper explores the issues of determining the spatial orientation of coordinate systems, the resulting slope and coordinate transformation with the application of Gobbs vectors.