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MATHEMATICAL PROCESSING OF RESULTS OF DOUBLE UNEQUALLY ACCURATE MEASUREMENTS

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Statement of the problem

In the practice of geodetic measurements there are cases of double unequally inaccurate measurements. Course books and reference books quote the following cases: first, double measurements are not mutually equally accurate, while they are equally accurate in each pair. And second, double measurements are unequally accurate both mutually and in each pair [1 – 6, 8, 9, 12, 13].

The analysis of double unequally accurate measurements shows that there is a case of double measurements that are mutually equally accurate, while being unequally accurate in each pair. That is, if the same values are obtained with measuring devices differing in accuracy. For instance, horizontal angles of theodolite traverse are measured twice with theodolites or electronic tachometers that are different in accuracy.

Scientific papers and course books state that systematic error is calculated, and then it is subtracted from differences without taking into account the weight of these differences. It is not subtracted from mean values of double unequally accurate measurements of each pair.

The given different criteria of calculating the value of systematic error do not allow reaching the same conclusion, i.e. the obtained results of calculation often suggest the opposite conclusions.

Calculation of confidence probability, which is essential to determine Student coefficient, is not entirely grounded.

It is obvious that the solution of the raised problems will significantly improve mathematical treatment of the results of double unequally accurate measurements and increase their accuracy.

Review of recent research and publications

Course books and scientific papers [1 – 6, 8, 12] point out that under double unequally inaccurate measurements, in cases of systematic error significance, it should be in fact calculated by the formula of general arithmetic mean value:

$$\theta = \frac{\Sigma pd}{\Sigma p}, \qquad (1)$$

and then it is subtracted from each difference:

$$d_i' = d_i - \theta, \qquad (2)$$

where θ is the systematic error in double unequally accurate measurements.

This approach of excluding systematic error is possible only in the case when it is equal in every unequally accurate difference, which is pointed out in [5].

In the first case of double unequally accurate measurements the mean value in the pair is calculated by the formula of simple arithmetic mean, while in the second case – by the formula of general arithmetic mean

$$\bar{x}_i = \frac{x_i p_i + x_i' p_i'}{p_i + p_i'}. \qquad (3)$$

The authors [6] fairly note that every difference should be calculated separately taking into account the conditions of every concrete problem (with the weights of every difference and the sum of all weights taken into account), in particular:

$$d_i' = d_i - \frac{\Sigma d}{\Sigma k} k_i \qquad (4)$$

or

$$d_i' = d_i - \frac{\Sigma d}{\Sigma S} S_i, \qquad (5)$$

where $\frac{\Sigma d}{\Sigma k}$ i $\frac{\Sigma d}{\Sigma S}$ is the systematic influence factor.

A. Chebotarev [13] takes into account the number of stations while subtracting systematic error from differences in double unequally accurate measurements.

The purpose of article

The above considerations suggest that the purpose of the article is to offer more accurate calculation of systematic error and its significance, and to subtract it not only from differences, but also from mean values of double unequally accurate measurements; to determine the necessary criterion of systematic error significance and to determine the choice of the value of confidence probability.

The main material

The calculations showed that using formulae (4) and (5) the total of summands of differences with systematic error subtracted multiplied by the square root of the weights of these differences can be not very big, but it never equals zero. Considering that the measurements are unequally accurate, it is necessary for the condition to be fulfilled after subtracting systematic error:

$$\Sigma \sqrt{p_d}\, d' = 0. \qquad (6)$$

In the first case of double unequally accurate measurements with significant systematic error, it is offered to calculate it by the formula:

$$\theta = \frac{\Sigma d\sqrt{p_d}}{\Sigma k\sqrt{p_d}} \qquad (7)$$

and then to subtract it from each difference:

$$d_i' = d_i - k_i\theta \qquad (8)$$

or

$$d_i' = d_i - S_i\theta\,, \qquad (9)$$

with control:

$$\Sigma d' = \Sigma d - \theta\Sigma k \qquad (10)$$

or

$$\Sigma d' = \Sigma d - \theta\Sigma S\,. \qquad (11)$$

It is offered to subtract systematic error from the mean values of each pair of measurements by the formulae:

$$\bar{x}_{i_{випр}} = \bar{x}_i - k_i\frac{\theta}{2} \qquad (12)$$

or

$$\bar{x}_{i_{випр}} = \bar{x}_i - S_i\frac{\theta}{2}\,. \qquad (13)$$

For the first case we give the example of mathematical treatment of the results of double unequally accurate measurements with significant systematic error (Table 1).

Table 1

Output information and calculation results in the first case

#	Cumulative value of elevations, mm		Differences d, mm	Number of stations k	Weights of measurements p	Weight of differences p_d	Average elevations $\bar{h}$, mm	Differences d', mm	Systematic error $k\,\theta$, mm	Corrected elevations $\bar{h}_{випр}$, mm
	direct, h	reverse, h'								
1	1205	-1202	3	2	2.00	1.00	1203.5	2.03	0.97	1203.01
2	-1492	1496	4	4	1.00	0.50	-1494.0	2.06	1.94	-1494.97
3	1275	-1272	3	2	2.00	1.00	1273.5	2.03	0.97	1273.01
4	1512	-1515	-3	3	1.33	0.67	1513.5	-4.46	1.46	1512.77
5	-1490	1487	-3	5	0.80	0.40	-1488.5	-5.43	2.43	-1489.72
6	1717	-1713	4	4	1.00	0.50	1715.0	2.06	1.94	1714.03
7	-1853	1855	2	2	2.00	1.00	-1854.0	1.03	0.97	-1854.49
8	1311	-1308	3	3	1.33	0.67	1309.5	1.54	1.46	1308.77
9	-1072	1068	-4	4	1.00	0.50	-1070.0	-5.94	1.94	-1070.97
10	-1100	1103	3	2	2.00	1.00	-1101.5	2.03	0.97	-1101.99
∑	13	-1	12	31	14.47	7.23	7.00	-3.07	15.07	-0.53

It should be noted that the values in some cells of Tables 1-3 are rounded off to the hundredth, however in calculations, more accurate values were used.

At the same time, the value of systematic error equals θ = 0.49 mm, $\Sigma\sqrt{p_d}\,d' = 0.00$ mm, and root mean square error of the unit of weight is μ = 2.49 mm calculated by the formula:

$$\mu = \sqrt{\frac{\Sigma p_d d'^2}{n}}\,. \qquad (14)$$

In formula (14) it is necessary to take the number of measurements n, rather than (n – 1). As d' are corrected differences, i.e. differences (random inaccuracy) with systematic error subtracted from it. However, it does not matter how this was made.

It is offered to calculate the second case of double unequally accurate measurements with significant systematic error by the formula (7). It should be subtracted from each difference by formulae (8) and (9), at the same time the greater values of k_i or S_i is chosen in each pair.

Then it is offered to subtract systematic error from mean values of each pair of measurements by the formulae:

$$\bar{x}_{i_{випр}} = \bar{x}_i - \frac{k_i\theta\,p}{p_i + p_i'} \qquad (15)$$

or

$$\bar{x}_{i_{випр}} = \bar{x}_i - \frac{S_i\theta\,p}{p_i + p_i'}\,, \qquad (16)$$

whereby, in each pair the lesser value of the two weights p is taken in the numerator.

The weight of each difference is calculated by the well-known formula:

$$p_{d_i} = \frac{p_i \cdot p_i'}{p_i + p_i'}\,. \qquad (17)$$

To illustrate the second case, we give the example of mathematical treatment of the results of double unequally accurate measurements with significant systematic error (Table 2).

Table 2

Output information and calculation results in the second case

#	Cumulative values of elevations, mm		Differences d, мм	Number of stations		Weights		Weights of differences p_d	Average elevations $\bar{h}$, mm	Systematic error $k\ \theta$, mm	Corrected elevations $\bar{h}_{виnp}$, mm
	direct, h	reverse, h'		direct, k	reverse, k'	direct, p	reverse, p'				
1	1205	-1202	3	2	2	2.00	2.00	1.00	1203.50	0.97	1203.02
2	-1492	1496	4	4	3	1.00	1.33	0.57	-1494.29	1.93	-1495.11
3	1275	-1272	3	2	1	2.00	4.00	1.33	1273.00	0.97	1272.68
4	1512	-1515	-3	3	2	1.33	2.00	0.80	1513.80	1.45	1513.22
5	-1490	1487	-3	5	4	0.80	1.00	0.44	-1488.33	2.42	-1489.41
6	1717	-1713	4	4	3	1.00	1.33	0.57	1714.71	1.93	1713.89
7	-1853	1855	2	2	1	2.00	4.00	1.33	-1854.33	0.97	-1854.66
8	1311	-1308	3	3	2	1.33	2.00	0.80	1309.20	1.45	1308.62
9	-1072	1068	-4	4	2	1.00	2.00	0.67	-1069.33	1.93	-1069.98
10	-1100	1103	3	2	1	2.00	4.00	1.33	-1102.00	0.97	-1102.32
∑	13	-1	12	31	21	14.47	23.67	8.85	5.93	14.98	-0.06

Whereby, the systematic error equals θ = 0.48 mm, $\Sigma\sqrt{p_d}\,d' = 0.00$ mm, and root mean square error of the weight unit is μ = 2.75 mm.

Further we will consider the third case of double unequally accurate measurements. The following conditions are fulfilled in such double unequally accurate measurements:

$$p_1 = p_2 = ... = p_n \text{ i } p_1' = p_2' = ... = p_n'. \qquad (18)$$

Whereby in each pair $p_i \neq p_i'$.

The weight of every difference is calculated in the same way as in the second case (17); however, here the weight of every difference is the same. Taking this into account, in case of signiticant systematic error it should be calculated by the formula of double equally accurate measurements and subtracted from each difference by the formula (2). It is offered to subtract systematic error from mean values of each pair of measurements by the formula similar to (15) or (16):

$$\bar{x}_{i_{виnp}} = \bar{x}_i - \frac{\theta\, p}{p_i + p_i'}. \qquad (19)$$

At the same time, in each pair the lesser value of the two weights p is taken in the numerator, as it is considered that the more accurate measurement must have lesser systematic error.

The mean values are calculated by the formula of general arithmetic mean (3), and root mean square error of the weight unit is calculated by the formula:

$$\mu = \sqrt{\frac{p_d \Sigma d'^2}{n}}. \qquad (20)$$

In formula (20), similarly to (14) it is necessary to take the number of measurements n, rather than (n – *1*).

Root mean square errors of average values will be equal to each other and be calculated by the formula:

$$m_{\bar{x}_i} = \frac{\mu}{\sqrt{p_i + p_i'}}. \qquad (21)$$

For the third case we give the example of mathematical treatment of the results of double unequally accurate measurements with significant systematic error (Table 3).

Whereby, the values of weights of each measurements of the first series equal p = 1, the second one is p' = 4, the weight of each difference is p_d = 0.8, systematic error is θ = 1", and root mean square error of the weight unit is μ = 3.82".

There exists the problem of unambiguous calculation of systematic error [7, 10]. Some course books [2 and oth.] offer the criteria of calculating systematic error, i.e. the cases when it can be neglected and when it should be subtracted in unequally accurate measurements:

$$\left|\Sigma d\sqrt{p_d}\right| \le 0.25\Sigma\left|d\sqrt{p_d}\right|, \qquad (22)$$

$$\left|\Sigma p_d d\right| \le 0.25\Sigma\left|p_d d\right|, \qquad (23)$$

$$\left|\Sigma p_d d\right| \le 1.25 t_\beta \frac{\Sigma\left|p_d d\right|}{\sqrt{\Sigma p_d}} \qquad (24)$$

or

$$\left|\Sigma p_d d\right| \le 2.5 \frac{\Sigma\left|p_d d\right|}{\sqrt{\Sigma p_d}}. \qquad (25)$$

Taking formula (22-24) into account, the following significance criterion of systematic error in double unequally accurate measurements is offered:

$$\left|\Sigma d\sqrt{p_d}\right| \le 1.25 t_\beta \frac{\Sigma\left|d\sqrt{p_d}\right|}{\sqrt{\Sigma\sqrt{p_d}}}. \qquad (26)$$

In formula (25) the factor 2.5 is obtained under the parameters n > 28 and β = 0.95. Thus, under different number of pairs and confidence probability, this formula is inappropriate to use, and it is advisable to use criteria (22-24) or (26).

Table 3

Output information and calculation results in the third case

#	Horizontal angle, ° ' "		Differences d, ° ' "	d^2	Differences d', ° ' "	d'^2	Mean value of the angle $\overline{\beta}$, ° ' "	Corrected value of the angle $\overline{\beta}$, ° ' "
	β_1	β_2						
1	101 20 33	101 20 27	6	36	5	25	28.2	28.0
2	182 37 45	182 37 40	5	25	4	16	41.0	40.8
3	213 41 15	213 41 21	-6	36	-7	49	19.8	19.6
4	95 07 09	95 07 12	-3	9	-4	16	11.4	11.2
5	121 14 05	121 13 59	6	36	5	25	0.2	0.0
6	147 50 14	147 50 18	-4	16	-5	25	17.2	17.0
7	221 08 30	221 08 25	5	25	4	16	26.0	25.8
8	189 25 16	189 25 14	2	4	1	1	14.4	14.2
9	105 52 39	105 52 38	1	1	0	0	38.2	38.0
10	61 43 01	61 43 03	-2	4	-3	9	2.6	2.4
∑	1440 00 27	1444 00 17	10	192	0	182	19.0	17.0

If in formulae (22 – 24) we accept that the weights of differences equal a unit (p=1), then we obtain the formulae of criteria given for double equally accurate measurements.

For the third case of double unequally accurate measurements, taking into account that the weight of all differences will be the same and not equaling a unit, the significance factor of systematic error (22) and (23) will turn into the expression of criterion given for double equally accurate measurements, in particular:

$$\left|\Sigma d\right| \le 0.25\Sigma\left|d\right| . \qquad (27)$$

Criterion (24) turns into the following expression:

$$\left|\Sigma d\right| \le 1.25 t_\beta \frac{\Sigma\left|d\right|}{\sqrt{np_d}} , \qquad (28)$$

and criterion (26) – into the folloing expression:

$$\left|\Sigma d\right| \le 1.25 t_\beta \frac{\Sigma\left|d\right|}{\sqrt{n\sqrt{p_d}}} . \qquad (29)$$

Let us give the results of calculation of criteria (22-24 and 26) by the data from Table 1 and randomly taken value of confidence probability β = 0.95:

$11.93 > 0.25 \cdot 26.28$ or $11.93 > 6.57$;

$11.80 > 0.25 \cdot 22.20$ or $11.80 > 5.55$;

$11.80 < 1.25 \cdot 2.3 \cdot 22.20 / \sqrt{7.23}$ or $11.80 < 23.74$;

$11.93 < 1.25 \cdot 2.3 \cdot 26.28 / \sqrt{8.39}$ or $11.93 < 26.08$.

Analyzing the obtained results of using criteria of significance of systematic error (22-24 and 26) for the first case of double unequally accurate measurements (Table 1), we can draw a conclusion that according to these criteria the opposite results are obtained. In accordance with criteria (22 and 23), systematic error cannot be neglected, and under criteria (24 and 26), we can netlect it. Obviously, this is unacceptable and only one criterion can be used. In this case the same results of the calculation will be obtained, which means that the choice of this or that criterion should be grounded.

Assume that all differences in Table 1 are positive, for example. We will use the criteria of calculating the significance of systematic errors (22-24 and 26) and obtain the following results:

$26.28 > 0.25 \cdot 26.28$ or $26.28 > 6.57$;

$22.20 > 0.25 \cdot 22.20$ or $22.22 > 5.55$;

$22.20 < 1.25 \cdot 2.3 \cdot 22.20 / \sqrt{7.23}$ or $22.20 < 23.74$;

$26.28 > 1.25 \cdot 2.3 \cdot 26.28 / \sqrt{8.39}$ or $26.28 > 26.08$.

Therefore, according to criterion (24), systematic error can be neglected, which is incorrect, as simple speculations show the necessity of subtracting systematic error. Other criteria establish the necessity of subtracting systematic error. This example again stresses the necessity of confidence probability. Obviously, one can assume that it is necessary to use criterion (22).

As mathematical expectation of true errors of double measurements tends to zero, taking maximum values of positive and negative differences with subtracted systematic error, beyond the limits of the interval of true errors (differences) occurrence, we can determine the probability of double measurements differences occurring in this interval, i.e. $d_{\min} \le D \le d_{\max}$, based on the examples in [4, 11].

According to Table 1, minimal value of difference equals d'_9 = –5.94 mm with the weight $p_{d_9} = 0.50$, while there are two maximum value differences $d'_2 = d'_6$ = +2.06 mm with the weights $p_{d_2} = p_{d_6} = 0.50$. In this example the weights of positive maximum differences are equal. However, it is more common when they have difference weights. The question is which difference with which weight should be taken for calculation of confidence probability? The findings show that it is necessary to use maximum values of the weights of differences. From the calculations it follows that confidence probability in intervals from $-t_{\min}$ = –1.69 to $+t_{\max}$ = +0.58 will be $\beta \approx 0.67$.

Taking into account the determined value of confidence probability β = 0.67, let's check the significance criteria of systematic error in double unequally accurate measurements (24) and (26):

$$11.80 > 1.25 \cdot 1.03 \cdot 22.20 / \sqrt{7.23} \text{ or } 11.80 > 10.63;$$

$$11.93 > 1.25 \cdot 1.03 \cdot 26.28 / \sqrt{8.39} \text{ or } 11.93 > 11.68.$$

The obtained results suggest that systematic error should not be neglected, although left-hand and right-hand sides of the equation are not so significantly different as under confidence probability equalling β = 0.95. That is, the calculated confidence probability β = 0.67 is more appropriate for this series of double unequally accurate measurements.

The results of calculating the systematic error by the significance criteria (22 – 24) and (26) and confidence probability according to the data from Table 2 in the second case of double unequally accurate measurements are similar to the results of the first case.

Comparing criteria (22 – 24) and (26), one can see that left-hand side of the pairs of expressions (22) and (26), and (23) and (24) are the same. Thus, we can establish the conditions under which these criteria can lead to similar conclusions concerning the significance of systematic error. First, let's equal the right-hand sides of expressions (22) and (26), i.e.:

$$0.25\Sigma\left|d\sqrt{p_d}\right| = 1.25 t_\beta \frac{\Sigma\left|d\sqrt{p_d}\right|}{\sqrt{\Sigma\sqrt{p_d}}} \qquad (30)$$

or

$$0.25 = 1.25 t_\beta \frac{1}{\sqrt{\Sigma\sqrt{p_d}}}\ . \qquad (31)$$

Whereby, after transformations

$$\sqrt{\Sigma\sqrt{p_d}} = 5 t_\beta\ . \qquad (32)$$

From equation (32) follows inequation:

$$\sqrt{\Sigma\sqrt{p_d}} \geq 5 t_\beta \qquad (33)$$

and the conclusion that if square root of the sum of square roots of weights of differences is equal or more than five times Student coefficient, then conclusions concerning the significance of systematic error by criteria (22) and (26) will coincide.

Then we equate the right-hand sides of the expressions (23) and (24), i.e.:

$$0...25\Sigma\left|p_d d\right| = 1.25 t_\beta \frac{\Sigma\left|p_d d\right|}{\sqrt{\Sigma p_d}}\ . \qquad (34)$$

Transforming expression (34) we can write:

$$\sqrt{\Sigma p_d} = 5 t_\beta \qquad (35)$$

and

$$\sqrt{\Sigma p_d} \geq 5 t_\beta\ . \qquad (36)$$

From the expressions (35) and (36) we can draw the similar conclusion to the one from the expressions (32) and (33). If square root of the sum of weights of differences is equal or more than five times Student coefficient, then conclusions concerning the significance of systematic error by criteria (23) and (24) will coincide. However, it should be noted here, that in spite of the different significances of weights of double unequally accurate measurements, conditions (33) and (36) can occur only under a large number of measurements.

All the above mentioned allows us to give preference to criterion (22). However, this does not mean that all other criteria can be neglected. On the contrary, they should be used for joint analysis and additional research of the obtained results of double unequally accurate measurements.

Now we use the given criteria of systematic error significance for the third case of double unequally accurate measurements (Table 3) with rather acceptable value of confidence probability β = 0.95:

$$10 = 0.25 \cdot 40 \text{ or } 10 = 10;$$

$$10 < 1.25 \cdot 2.3 \cdot 40 / \sqrt{10 \cdot 0.8} \text{ or } 10 < 40.66;$$

$$10 < 1.25 \cdot 2.3 \cdot 40 / \sqrt{10\sqrt{0.8}} \text{ or } 10 < 38.45.$$

According to the calculation results, left-hand and right-hand sides of the criterion of systematic error significance (27) are equal, that's why, just like in case with double equally accurate measurements, in this case systematic error should be subtracted. Criteria (28) and (29) unambiguously indicate that systematic error can be neglected; however simple considerations bring to the opposite conclusions. That's why in the third case of double unequally accurate measurements it is worthwhile to use only criterion (27).

For the third example (Table 3) m_d = 4.27". Therefore confidence probability in the intervals from $-t$ = –1.64 to $+t$ = +1.17 will be β = 0.83.

Taking the calculated value of confidence probability β = 0.83, let's check the criteria of systematic error significance in double unequally accurate measurements (28) and (29):

$$10 < 1.25 \cdot 1.52 \cdot 40 / \sqrt{10 \cdot 0.8} \text{ or } 10 < 26.87;$$

$$10 < 1.25 \cdot 1.52 \cdot 40 / \sqrt{10\sqrt{0.8}} \text{ or } 10 < 25.41.$$

According to the calculated results, we can draw a conclusion that systematic error can be neglected, however, left-hand and right-hand sides of inequations do not differ so significantly as with confidence probability taken as β = 0.95. That is, the calculated confidence probability β = 0.83 is more appropriate to this series of double unequally accurate measurements.

The given examples (Tables 1 – 3) and many other experimental calculations that are not mentioned in this paper, show that after subtracting systematic error from mean values of double unequally accurate measurements of angles and elevations, the value of residual drops.

In unlimited variety of joint double unequally accurate measurements it happens that after subtracting systematic error from differences, the sum $\Sigma\sqrt{p_d}d$ can equal nonzero and divert by some value. This indicates that the used formulae do not take into account the true mechanism of mutual influence of systematic and random errors. In this case it is necessary to carry out additional research of the conditions and results of double unequally accurate measurements. There is a possibility that this is not an influence of systematic error but a manifestation of interaction of values of random errors in each pair of double unequally accurate measurements in their total value.

Conclusions

1. It is established that the criteria of systematic error significance in double unequally accurate measurements (22 – 25) given in previous researches, in fact lead to the opposite conclusions. To define the significance of a systematic error it is offered to use only criterion (22). Other criteria could be used only to analyze the results of double unequally accurate measurements and their study.

2. The formulae for subtracting systematic errors from differences and mean values of measured double values are offered for the two cases of double unequally accurate measurements given in the previous researches. Whereby, results of accuracy estimation correspond to the mean values of the measured values, while the values of residual in angle measurements and elevations decrease.

3. There is established the third case of unequally accurate measurements. For this case, there are given formulae of calculating systematic error, subtracting it from the difference and mean values of double unequally accurate measurements and estimating their accuracy.

4. To calculate root mean square error of the weight unit (after subtracting systematic error from the differences), one should use formulae (14) or (20) depending on the case of double unequally accurate measurements, i.e. square root of the sum of squares of corrected differences divided by the number of double unequally accurate measurements.

5. To study the differences of double equally accurate measurements it is necessary to calculate confidence probability, however in some cases it could be accepted based on certain considerations.

6. There is made a speculative assumption: if the absolute total of the sum of differences of double equally accurate measurements by the square root of their weights is located within the interval between zero and half of the total of the sum of their absolute values, then it is possible that it is not the result of a systematic error but it is a manifestation of the joint action of random errors.

References

1. Bol'shakov V.D. Teorija oshibok nabljudenij [*The theory of errors of observation*]: ucheb. dlja vuzov. – 2-e izd., pererab. i dop. / V.D. Bol'shakov. – M.: Nedra, 1983. – 223 s.
2. Bol'shakov V.D. Uravnivanie geodezicheskih postroenij [*Adjustment of geodetic network*]: spravochnoe posobie / V.D. Bol'shakov, Ju.I. Markuze, V.V. Golubev. – M.: Nedra, 1989. – 413 s.
3. Viduev N.G. Verojatnostno-statisticheskij analiz pogreshnostej izmerenij [*Probabilistic and statistical analysis of measurement errors*] / N.G. Viduev, G.S. Kondra. – M.: Nedra, 1969. – 320 s.
4. Voitenko S.P. Matematychna obrobka heodezychnykh vymiriv. Teoriia pokhybok vymiriv [*Mathematical processing of geodetic measurements. Theory of measurement errors*]: navchalnyi posibnyk / S.P. Voitenko. – K.: – KNUBA, 2003. – 216 s.
5. Gajdaev P.A. Teorija matematicheskoj obrabotki geodezicheskih izmerenij [*The theory of mathematical processing of geodetic measurements*] / P.A. Gajdaev, V.D. Bol'shakov. – M.: Nedra, 1969. – 400 s.
6. Zazuliak P.M. Osnovy matematychnoho opratsiuvannia heodezychnykh vymiriuvan [*Basics of mathematical processing of geodetic measurements*]: navchalnyi posibnyk / P.M. Zazuliak, V.I. Havrysh, E.M. Yevsieieva, M.D. Yosypchuk. – L.: Vyd-vo "Rastr-7", 2007. – 408 s.
7. Ivina D.S. Porivniannia kryteriiv znachymosti systematychnoi pokhybky pry otsintsi tochnosti za riznytsiamy podviinykh rivnotochnykh vymiriv [*Comparison of criteria of significance of systematic error in assessing the accuracy of measuring differences of double unuqually accurate measurements*] / D.S. Ivina, V.V. Riabchii // Tezy dopovidei KhIII Mizhnarodnoi naukovo-praktychnoi konferentsii molodykh uchenykh i studentiv «Polit. Suchasni problemy nauky», Kyiv, 3-4 kvitnia 2013 r. – S. 319.
8. Mazmishvili A.I. Teorija oshibok i metod naimen'shih kvadratov [*Theory of errors and the method of least squares*] / A.I. Mazmishvili. – M.: Nedra, 1978. – 311 s.
9. Papazov M.G. Teorija oshibok i sposob naimen'shih kvadratov [*Theory of errors and the method of least squares*] / M.G. Papazov, S.G. Mogil'nyj. – M.: Nedra, 1968. – 302 s.
10. Rjabchij V.A. Sravnenie kriteriev znachimosti sistematicheskoj oshibki pri ocenke tochnosti po raznostjam dvojnyh ravnotochnyh izmerenij [*Comparison of tests of significance of systematic error in the assessment of accuracy by differences of double equally accurate measurements*] / V.A. Rjabchij, D.S. Ivina // Zbirnyk prats IV Vseukrainskoi naukovo-tekhnichnoi konferentsii studentiv, aspirantiv i molodykh vchenykh «Naukova vesna 2013», Dnipropetrovsk, 28-29 bereznia 2013 r. – S. 176-177.
11. Riabchii V.A. Ymovirno-matematychnyi analiz obmezhenoi kilkosti rezultativ nerivnotochnykh vymiriv odniiei velychyny [*Probabilistic and mathematical analysis of limited number of measurement results of unuqually accurate*

measurements of one value] / V.A. Riabchii, V.V. Riabchii // Suchasni dosiahnennia heodezychnoi nauky ta vyrobnytstva: zb. nauk. pr. Zakh. heodez. t-va UTHK. – L.: Vyd-vo Lvivskoi politekhniky, 2013. – Vyp. II. – S. 25-30.

12. Riabchii V.A. Teoriia pokhybok vymiriuvan [*Theory of measurement errors*]: navchalnyi posibnyk / V.A. Riabchii, V.V. Riabchii. – Dn-sk: Nats. hirn. un-t, 2006. – 166 s.
13. Chebotarev A.S. Sposob naimen'shih kvadratov s osnovami teorii verojatnostej [*The method of least squares with the basics of probability theory*]: Uchebnik dlja geodezicheskih vuzov i fakul'tetov / A.S. Chebotarev. – M.: Geodezizdat, 1958. – 606 s.

Mathematical processing of results of double unequally accurate measurements

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The existing information on the results of mathematical processing of double unequally measurements provided in scientific and academic literature is analyzed. Irregularities of the conclusion on the criteria of systematic error significance are pointed out. The research results provide an opportunity to choose the criterion of systematic erro significance. Some of these criteria are proposed to use only for research and measurement differences, and thus to calculate the confidence probability. The method of systematic error calculation is offered based on weights of measurements and systematic error subtraction from the differences and the mean values of measured double unequally accurate measurements. Application of this method improves accuracy estimation and reduces residual.