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ESTIMATION OF EARTH SURFACE DEFORMATIONS ACCORDING TO THE DATA IN GEODETIC CURVILINEAR COORDINATE SYSTEMS

A. Tadyeyev
National University of Water Menegment and Nature Resource Use, Rivne

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Statement of the problem and its connection with important scientific and practical tasks

Estimation the earth surface deformations are one of the main problems of modern geodynamics. The solution to the problem is achieved by methods of geodesy, geophysics, geotectonic and other earth sciences. Objects of their applied research make up a single dynamic system. Therefore, indicators of versatile monitoring of the Earth, comparing and search relationships the force fields data of different physical origins, interdisciplinary cooperation and complex interpretation of the final results make it possible not only to determine the deformed state of the earth's crust and its surface, but also to reveal the genesis of geodynamic processes and study the evolution of the Earth as a whole. From the standpoint to satisfy of public needs this ensures the risk assessment in terms of life safety and prognosis of probable catastrophic consequences. In the applied sense the synthesis of research results of various natural sciences to solve of geodynamic problems is a necessary condition of strict solution the traditional and new problems of physical geodesy, such as reduction of measurements in a single reference system of coordinates and gravity.

Methods of geodetic monitoring of geodynamic processes, as the main source of quantitative expression their spatial and time structure, includes repeated observations of geodetic networks, the data processing and interpretation of results. The formation of a powerful database of observations have been provided by active development and introduction in the geodetic production the modern satellite navigation technologies, that are implemented by global networks of permanent GNSS-stations. Satellite tools of the Earth surface positioning capable of guaranteeing the definition of representative displacements indicators of observation stations. These data make it possible to estimate the recent earth surface movements of different scales and reflect them by various cartographic models; to describe the dynamics of lithosphere plates and more. However, the theoretical basis and methods of data processing in order to determine the deformation of the surface and next analysis of results remains practically unchanged over the last decades. Based on current realities, indicated task requires, of at least, improved solutions. Analysis of the essence, the shortcomings of traditional methods and some prospects of alternative approaches to solving the problem are disclosed in article [6].

Analysis of the research and unresolved parts of the general problem

The earth's surface movements that are expressed numerically by the results of repeated geodetic observations can be identified as the transformation of the Earth surface, which is reduced to one or another pertinence reference surface. Any of the conventional geodetic reference surfaces has a geometric essence and leads to establish of an appropriate coordinate system for geodetic stations then should be observed. This motivation allows us to consider the problem from a geometric viewpoint without regard to origin and nature of earth's surface movements. In articles [6, 7] the problem outlined from the standpoint of projective-differential geometry. Theoretical basis and legitimacy of the chosen approach are substantiated. To search for ways of solving the problem to deformations estimating applied the theory of surfaces [2] and some decision concerning taking into account of distortions at their reflection [3].

In article [7] defined the ways of solving the problem on the basis of the general theory of reflections; solutions are systematized in typical modeling geodetic coordinate systems with transfer of data processing to appropriate reference surfaces. Results are coordinated to terms of the surfaces parameterization of general theory of reflections. The analysis of the content of distortions characteristics of surfaces in their reflection (deformation characteristics) and their comparison with the corresponding analogues, which operate in solving geodynamic problems on the basis of the classical theory of continuous medium deformation [1] have been conduct. The systematization takes into account the prospects of the application of data processing results for the interpretation of geodynamic processes of different scales. Also, a general algorithm for solving the problem is defined in this article.

The solving of the problem on the plane in a rectangular coordinate system, which is based on the theory of quasiconformal surfaces reflections with Riemann parameterization, is disclosed in article [4]. It is proved that if the reflection is implemented by functions of the linear form, which is the basis of an affine transformation of rectangular coordinates in the linear theory of continuous medium deformations, the accuracy of the proposed solution is higher. The basic advantage of this solution is the ability to solve the problem without regard to the analytical form of reflection functions in order to only they satisfy a homeomorphism conditions.

In article [5] made the first attempt to solve the problem of estimation of earth surface deformations in curvilinear coordinate systems. The solving in a spherical geocentric system with transfer of data processing on the geosphere has been disclosed. This model traditionally used in the description of Earth force fields data in physical geodesy. If it is applied to solve the problem and describe the field of earth surface deformations by rows of spherical functions, eventually disclosed prospects for finding of interrelations of data fields with different physical meaning within the same model. This simplifies the compatible interpretation of results for the needs of geodynamics and to study of the Earth's figure.

Geosphere is the simplest curvilinear geodesic reference surface. In practice, for solving of geodetic tasks more commonly used another mathematically correct surface that corresponds to the model of earth ellipsoid of revolution with geodetic coordinate system. Therefore represents the corresponding interest the solving of problem the deformations estimation of the earth surface which is transferred to the ellipsoid.

Formulation of the problem

The aim is to express the parameters of deformation of the earth's surface, which is transferred to the earth ellipsoid of revolution, and to compare results with similar in the geosphere. The theoretical basis of the solution is a general theory the reflection of surfaces of revolution with isometric parameterization. Based on the fact that the spherical and geodetic coordinate systems are similar to isometric, the process of solving the problem is expected identical to opened in article [5].

The main material of researches

The general algorithm of solving the problem contains two blocks of tasks [7].

The first block forms the task of approximation the functions by the empirical distribution of observable coordinates of stations in order to determine the law of surfaces reflection. According to the general theory [2], if the isometric surface reflected on isometric to her surface, the class of functions that have implemented such reflection is the harmonic type. These functions are endowed the property of homeomorphism that makes impossible the strict solution of task in terms of requirements the theory of surfaces. Therefore, decision making concerning the choice of analytical forms of functions and identifying appropriate empirical formulas need to substantiate in terms of the correctness of his formulation, or content of solvable problem, or formally from indicators of the approximation accuracy.

Tasks of the second block aimed at the expression and accuracy estimating parameters of the earth's surface deformation, using installed at the previous stage empirical formulas. In terms of objectives of this article it is major decisions.

The basis for the formation of the deformation field and expression its parameters is a reflection tensor. This is a symmetric coefficients matrix of the first quadratic (metrical) form that corresponds to the linear elements ds' on the reflection surface S' and expressed by the linear element ds of the initial (nondeformed) surface S. Matrix has a constant structure and formation algorithm for all surfaces and completely dependent on reflection functions. Its components are partial derivatives functions of coordinates of the deformed surface from its initial coordinates and express the transformation, which exposed coefficients of the initial surface metrical form during the differentiated transformation of coordinates. Tensor applies to a separate point on the surface. If defines the tensor for the whole region of reflection, then formed tensor field that defined the deformation field with his characteristic parameters [7].

The general theory examines surfaces of the revolution in the curvilinear coordinate system of isometric latitude q and longitude λ. In isometric parameterization the linear surface element is expressed by the formula

$$ds^2 = r^2(dq^2 + d\lambda^2),$$

where r - the radius of the parallel to latitude q [2].

Geodetic tasks in the geosphere are solved in the spherical geocentric coordinate system of latitude φ (or codeclinations $\theta = \pi/2 - \varphi$) and longitude λ. The geosphere has a constant curvature, which expresses the radius R. Considering the interrelation between differentials of isometric and spherical latitudes

$$dq = \frac{R}{r}d\varphi = \frac{d\varphi}{\cos\varphi} = \frac{R}{r}d\theta = \frac{d\theta}{\sin\theta},$$

the linear element of geosphere is expressed by the formula

$$ds^2 = r^2\left(\left(\frac{R}{r}\right)^2 d\theta^2 + d\lambda^2\right). \qquad (1)$$

$r = R\cos\varphi = R\sin\theta$ - the radius of the parallel to latitude φ [5].

The most optimal for solving tasks on the ellipsoid is a geodetic coordinate system of latitude B and longitude L. Curvature of the surface at any point of the ellipsoid defined by two major normal radiuses: the radius of curvature in the meridian section M and the radius of curvature in the prime vertical section N. Considering interrelation

$$dq = \frac{M}{r}dB,$$

where $r = N\cos B$ - the radius of the parallel to latitude B, the linear element of ellipsoid is expressed by the formula

$$ds^2 = r^2\left(\left(\frac{M}{r}\right)^2 dB^2 + dL^2\right). \qquad (2)$$

The quantities that expressed by formulas (1) or (2) are metric measures for corresponding to them surfaces.

We distinguish on surface S the locked continuous region Δ that defines a finite number of geodetic points with coordinates θ,λ or B,L. If the earths surface for whatever reasons is deformed, the position of the same of points will determine the coordinates θ',λ' or B',L'. Then the region $\Delta \subset S$ is transformed into the region Δ' of the surface S'. On condition that kept the property of locked continuous region for Δ', the initial surface S is reflected into the surface S'. If dependences between points coordinates θ,λ and θ',λ' (or between B,L and B',L') is expressed by functions

$$\left.\begin{aligned}\theta' &= u(\theta,\lambda)\\ \lambda' &= v(\theta,\lambda)\end{aligned}\right\}$$

or

$$\left.\begin{aligned}B' &= u(B,L)\\ L' &= v(B,L)\end{aligned}\right\} \qquad (3)$$

and such functions have the homeomorphism property, then they, along with metric measures (1) or (2) allow to describe the change in internal geometry of the reflection region and thus to form a deformation field with his characteristic parameters. The solution of such task, which is attributed to the geosphere, is described in the article [5]. On this basis, we solve the task in Earth ellipsoid, and then compare the results.

To form a deformation tensor the metric measure of the reflection surface S', e.g.

$$ds'^2 = r'^2\left(\left(\frac{M'}{r'}\right)^2 dB'^2 + dL'^2\right), \qquad (4)$$

we expresses by coordinates of the initial surface S. Considering the change of variable $q = q(B)$, reflection functions of surfaces with isometric parameterization

$$\left.\begin{aligned}q' &= u(q,\lambda)\\ \lambda' &= v(q,\lambda)\end{aligned}\right\}$$

we transforms at geodetic system:

$$\left.\begin{aligned}q'(B') &= u(q(B),L)\\ L' &= v(q(B),L)\end{aligned}\right\}. \qquad (5)$$

Now we will express full differentials of functions (5):

$$\frac{M'}{r'}dB' = \frac{\partial u}{\partial B}dB + \frac{\partial u}{\partial L}dL;$$

$$dL' = \frac{\partial v}{\partial B}dB + \frac{\partial v}{\partial L}dL.$$

Substituting them into the metric measure (4) eventually gives the formula

$$ds'^2 = r'^2(edB^2 + 2fdBdL + gdL^2),$$

where

$$\left.\begin{aligned}e &= \left(\frac{\partial u}{\partial B}\right)^2 + \left(\frac{\partial v}{\partial B}\right)^2\\ f &= \frac{\partial u}{\partial B}\frac{\partial u}{\partial L} + \frac{\partial v}{\partial B}\frac{\partial v}{\partial L}\\ g &= \left(\frac{\partial u}{\partial L}\right)^2 + \left(\frac{\partial v}{\partial L}\right)^2\end{aligned}\right\}. \qquad (6)$$

Coefficients (6) form a transformation tensor of metric measure of the initial surface because of its reflection (deformation tensor):

$$\begin{pmatrix} e & f\\ f & g\end{pmatrix}. \qquad (7)$$

Its determinant

$$h^2 = eg - f^2 = \left(\frac{\partial u}{\partial B}\frac{\partial v}{\partial L} - \frac{\partial u}{\partial L}\frac{\partial v}{\partial B}\right)^2.$$

The general theory of surfaces imposes the condition: $h^2 > 0$. Since this condition can be violated in ellipsoid poles, these points are not to be taken into account at processing data.

Tensor (7) defines the deformation field of the region $\Delta \subset S$. Now we present formulas which express its parameters. By analogy with [5], divides the parameters into three groups depending on their content and purpose.

1. *Indicators linear distortions of the region* Δ. In general the scale of reflection that expressed by functions (3), is the result of relation of metric forms (4) and (2):

$$\mu^2 = \frac{ds'^2}{ds^2} = e\frac{r'^2}{M^2}\cos^2 A + f\frac{r'^2}{Mr}\sin 2A + g\frac{r'^2}{r^2}\sin^2 A.$$

Coefficient (or module) of the linear expansion μ characterizes the length change of the linear element ds in transition to ds' at the point B,L in the direction that indicates the azimuth

$$A = arctg\frac{rdL}{MdB}.$$

Solution of the equation $\frac{d(\mu^2)}{dA} = 0$ discloses main directions of distortions $A_\circ$ and $A_\circ + \pi/2$:

$$A_\circ = \frac{1}{2}arctg\frac{2fMr}{er^2 - gM^2}.$$

Main directions determine extreme scales of the reflection at the point of initial surface that are expressed by coefficients of maximum $\mu_{\max} = a$ and minimum $\mu_{\min} = b$ expansions:

$$a^2 = \frac{r'^2}{2M^2r^2}\left(er^2 + gM^2 + \sqrt{(er^2 - gM^2)^2 + 4f^2M^2r^2}\right);$$

$$b^2 = \frac{r'^2}{2M^2r^2}\left(er^2 + gM^2 - \sqrt{(er^2 - gM^2)^2 + 4f^2M^2r^2}\right).$$

Scales of the reflection in directions of meridians and parallels when $A=0$ and $A=\pi/2$ characterizes coefficients

$$\mu_0^2 = m^2 = \frac{r'^2}{M^2}e\ ;$$

$$\mu_{\pi/2}^2 = n^2 = \frac{r'^2}{r^2}g\ .$$

Parameters of the deformation field in the geosphere, which belong to this group, disclose the following formulas [5]:

$$\mu^2 = e\frac{r'^2}{R^2}\cos^2 A + f\frac{r'^2}{Rr}\sin 2A + g\frac{r'^2}{r^2}\sin^2 A;$$

$$A = arctg\frac{rd\lambda}{Rd\theta};$$

$$A_\circ = \frac{1}{2}arctg\frac{2fRr}{er^2 - gR^2};$$

$$a^2 = \frac{r'^2}{2R^2r^2}\left(er^2 + gR^2 + \sqrt{(er^2 - gR^2)^2 + 4f^2R^2r^2}\right);$$

$$b^2 = \frac{r'^2}{2R^2r^2}\left(er^2 + gR^2 - \sqrt{(er^2 - gR^2)^2 + 4f^2R^2r^2}\right);$$

$$m^2 = \frac{r'^2}{R^2}e\ ;$$

$$n^2 = \frac{r'^2}{r^2}g\ .$$

Taking into account the prospect of comparing the formulas for calculating the deformation field parameters in the geosphere and ellipsoid should pay attention to differences in the expression of the radius of parallels r to these surfaces.

2. *Indicators angular distortions of the coordinate lines system and the region* Δ. The general theory of reflection [2] allows by angular measures express distortions the system of coordinate lines which parameterized the surface. It is based on relations of differentials the projections of coordinate lines on the reflections surface that are expressed in the coordinate system of the initial surface. In order to obviously express such relations, is necessary to define functions that have implemented the reflection. On the surfaces of revolution the coordinate lines system formed meridians and parallels. Their network in an isometric parameterization coincides with the same network in spherical and geodetic parameterizations. Are different only the coordinate systems relative of meridians and parallels on various surfaces. They are installed for reasons the optimal description of the absolute position of points on one or another surface. Thus, the justification for the solution of this part of the task on the geosphere and ellipsoid are identical. Here is the final result of the solution - formulas to express various indicators of angular distortions of geodetic coordinate system. Therefore,

$$tg\psi = \left(\frac{\partial v}{\partial B}\right)\Big/\left(\frac{\partial u}{\partial B}\right);$$

$$tg\chi = \left(\frac{\partial v}{\partial L}\right)\Big/\left(\frac{\partial u}{\partial L}\right).$$

ψ and χ – azimuths projections of meridians and parallels of the initial surface during the transition to the surface of reflection according to the functions (3). If deformations of the initial surface, which are implemented by these functions, led to the orthogonal distortion of meridians and parallels on the surface of reflection, the difference

$$\vartheta = \chi - \psi$$

or angle

$$\varepsilon = \vartheta - \frac{\pi}{2}$$

are indicators of such distortion. These indicators are expressed by formulas

$$\vartheta = arctg\frac{h}{f}\ ;$$

$$\varepsilon = -arctg\frac{f}{h}\ .$$

Formulas for expression of indicators ϑ and ε on ellipsoid and geosphere are identical.

In terms of content parameters of this group, they can have dual practical application. The first thing is obvious - Earth's poles motion detection and together with indicators of linear distortions, coordinate systems deformations definition. In terms of solving geodynamic tasks a direct relation to the interpretation of the earth's surface deformation fields has the angle $\varepsilon/2$ which is an indicator of the rotation of the reflection region Δ.

3. *Indicators area distortion of the region* Δ. The scale of area expresses the ratio of differentials dp'/dp. $dp = MrdBdL$ – an elementary initial surface area around a given point, which is a region of coordinates B, L changes and corresponds to pairs of their values. $dp' = M'r'dB'dL'$ – an appropriate area on the reflection surface. The consequence of this ratio is the coefficient of relative change of area of the region Δ:

$$p = \frac{r'^2}{Mr}h\ . \qquad (8)$$

The formula (8) can also be obtained from the product

$$p = ab\ .$$

The coefficient of relative change of area of the reflection region on the geosphere expresses formula

$$p = \frac{r'^2}{Rr}h\ .$$

Conclusions and prospects of subsequent researches

1. The presented formulas allow estimating the earth's surface deformation based on observations that are expressed in geodetic coordinate system. Estimates of deformations attributed to the earth ellipsoid of revolution.

2. Method of determining the deformations of the earth's surface, which refers to the ellipsoid and methods [4] and [5], are based on the theory of surfaces reflection. Such an alternative approach to solving the problem significantly expands informative possibilities of geodetic monitoring methods of geodynamic processes concerning the deformation field's interpretation.

3. Comparison the formulas for calculating the deformation parameters, which belong to the ellipsoid and geosphere, showed such differences. First, a clear, is reduced to the take account of the curvature of surfaces with different parameterization. This difference applies equally to relevant calculation formulas for a plane [4], where the curvature is zero, and the function that it expresses, $\lambda(u,v)\equiv 1$. The second difference lies in components of the tensor which are expressed the deformation field parameters. A tensor is determined by reflection surfaces functions. It follows that this difference is also caused by parameterization of the surface and its curvature indicators.

4. Similar to previous conclusion formulated in the monograph [3], but on the results of other applied researches based on the theory of surfaces reflection. In particular, studies of optimal projections of different surfaces on a plane in mathematical cartography. Identity of conclusions confirms the reliability of our results.

5. The fact which witnessed in the previous rubrics, gives grounds to perform the transformation of deformation fields from a single reference geodetic surface to any other desired surface. This can be achieved by expression the interconnections of curvature parameters of different surfaces and their coordinate systems. Hypothetically, this is a simple mathematical task. Transformation of deformation fields has obvious prospect due to necessity of considering of geodynamic effects (e.g., deformation fields of planetary scale) for determining (closer definition) of the Earth's figure.

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A. Tadyeyev

The solution of the task of estimation the earth surface deformations by the measuring data which are expressed in the geodetic (ellipsoidal) coordinate system is disclosed. Expression of deformation parameters is based on the theory of the surfaces reflection. Results of the solution on the earth ellipsoid of revolution compared with similar results in the geosphere.