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DIFFERENTIAL FORMULAS OF THE FIRST GENUS FOR DETERMINATION OF COMMON SURVEY COMPUTATION IN THREE-DIMENSIONAL COORDINATE SYSTEM

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Problem statement

All points of modern geodetic control network determination or in three-dimensional geodetic coordinates of *B,L,H* or in three-dimensional geocentric coordinates of *X,Y,Z.* The values of these coordinates get by the decision of common survey computation between adjacent points. The initial sizes for this decision accept: geodetic coordinates of the initial point *B,L,H* (geocentric coordinates of *X,Y,Z*), azimuth bearing, distance between points and zenith distance of this direction. Under the action of global and regional geophysical processes there are changes of both coordinates of initial points and initial measurement parameters, that imply the change of coordinates of next points of network. As a result we wanted to get the differential formulas for the direct calculation of corrections in the coordinates of points due to the changes listed above, without the repeated decision of common survey computation.

Analysis of the last researches and publications

The points of the geodetic network set in one or other system of coordinates. Realization of the proper system of coordinates (for example, ITRF) on a certain epoch is provided the certain amount of earthly permanent points [1], which take into account dynamic descriptions of Earth. For this reason, the systems of coordinates change all the time and complemented with the data, leading to appearance of new realization of these systems, such as ITRF89, ITRF96, ITRF2000, ITRF2005, ITRF2008. It means that the coordinates of initial points of geodesic network will be constantly elaborated. Except for it, geodynamical horizontal and vertical movements of some other tectonic blocks imply the change of measurement elements in geodesic networks. Its testify that support of geodesic networks at technologically advanced is possible only by the permanent account of these influences.

One of the ways support of geodesic networks at technologically advanced is applications of differential formulas, which provide the calculation of corrections in existent coordinates without the repeated adjustment and measurement of coordinates. Differential formulas that used at the decision of the proper tasks on-the-spot of ellipsoid are considered in modern scientific and technical literature [2,3], and decision of the proper tasks in three-dimensional coordinates directly used of three-dimensional rectangular coordinates [2,3].

Problem definition

The primary purpose of work is development of differential formulas for the receipt of corrections in three-dimensional geodesic coordinates, which will take into account the changes of three-dimensional geodesic coordinates of initial point, and also changes of three-dimensional topocentric polar coordinate in a initial point.

Exposition of basic material

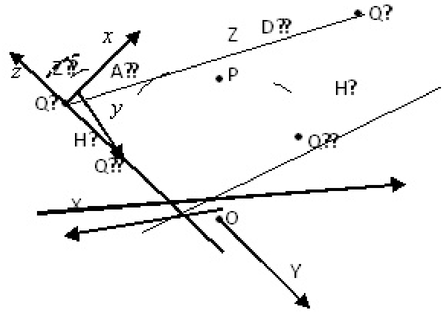
Main point of common survey computation in the three-dimensional coordinate systems consists in deciding three-dimensional geodesic coordinates of next point of space after the known three-dimensional geodesic coordinates of previous point and after the known spatial topocentric polar coordinate in this initial point.

Theoretical principles of receipt of differential formulas for the decision of common survey computation in the three-dimensional coordinate systems are based on the use of the known formulas of connection between geodesic and three-dimensional geocentric coordinates [3]

$$X = (N + H)\cos B \cos L,$$
$$Y = (N + H)\cos B \sin L,$$
$$Z = (N(1 - e^2) + H)\sin B \qquad (1)$$

and formulas that establish a connection between three-dimensional topocentric and three-dimensional polar coordinate in a observed point, fixed the origin of topocentric system coordinates (*x,y,z)* in this point (Fig.1).

X axis in the topocentric coordinate system is hold at a plane of geodesic meridian of point Q_1 and it positive direction is directed on a north, *z* axis – is external continuation of normal in the point of Q_1, and *y* axis in adds the system of coordinates to left. The three-dimensional polar coordinate system is set in the point Q_1 by the geodesic zenith distance Z_{12}, the geodesic azimuth of A_{12} and distance of D_1 to the next point of Q_2.



Geocentric and topocentric coordinate systems

Topocentric coordinates of point of Q_2 through polar coordinate are determined after formulas

$$x = D_{12} \sin Z_{12} \cos A_{12},$$
$$y = D_{12} \sin Z_{12} \sin A_{12},$$
$$z = D_{12} \cos Z_{12}, \qquad (2)$$

And after three-dimensional geocentric coordinate for these values we have[2]:

$$x = \left[N_2 \left(1 - e^2\right) + H_2 \right] \sin B_2 + e^2 N_1 \sin B_1$$
$$\cos B_1 - \left(N_2 + H_2\right) \cos B_2 \cos\left(L_2 - L_1\right) \sin B_1 ,$$
$$y = \left(N_2 + H_2\right) \cos B_2 \sin(L_2 - L_1),$$
$$z = \left[N_2 \left(1 - e^2\right) + H_2 \right] \sin B_2 + e^2 N_1 \sin B_1 +$$
$$\left(N_2 + H_2\right) \cos B_2 \cos(L_2 - L_1) \cos B_1 - (N_1 + H_1). \qquad (3)$$

Let three-dimensional geodetic and polar coordinate of initial point get infinitesimal values dB_1, dL_1, dH_1, dA_{12}, dD_{12}, dZ_{12}. It means that changes must take place in position on the ellipsoid of point of Q_2, and also its height variation. Variation in position of point Q_2 on-the-spot of ellipsoid determined the variation of geodesic coordinates on the size of corrections ΔB and ΔL. Final values of corrections ΔB_{2k} and ΔL_{2k} will be the result of total influence of two factors on the coordinates: first estimates influence on the geodesic coordinates of point Q_2 of variation of parameters which are used for determination of common survey computation on-the-spot of ellipsoid, namely variation of initial geodetic coordinates dB_1 and dL_1, variation value of initial azimuth dA_{12}, distance of geodetic line ds_{12} (dD_{12}); - and second estimates influence on the geodesic coordinates of point Q_2 of variation of parameters of three-dimensional polar coordinate dA_{12}, dD_{12} and dZ_{12}.

let us denote the corrections in the geodetic coordinate of point Q_2, as - $\Delta B'$ and $\Delta L'$. They define the influence of the first factor. The value of these corrections calculate after known differential formulas of the first genus for determination of common survey computation on-the-spot of ellipsoid. In scientific and technical literature [2,3] these formulas have a difficult and lengthy kind. For the calculation of the corrections with an accuracy to 0,0001″ these can be simplified and write down after transformation so:

$$\Delta B_2' == \frac{M_1}{M_2} \cos l \Delta B_1'' -$$
$$-\frac{\rho''}{M_2} \cos A_{21} \, \Delta s_{12} + \frac{m}{M_2} \sin A_{21} \, \Delta A_{12}'',$$
$$\Delta L_2' = \Delta L_1'' + \frac{M_1}{N_2} \tan B_2 \sin l \, \Delta B_1'' -$$
$$\frac{\rho''}{N_2} \sec B_2 \sin A_{21} \, \Delta s_{12} - \frac{m}{N_2} \sec B_2 \cos A_{21} \, \Delta A_{12}'' \qquad (4)$$

where $l = L_2 - L_1$, $m -$ equated mileage of geodetic line

$$m = R_1 \sin \frac{s_{12}}{R_1},$$

R_1 - mean radius of curvature of ellipsoid in an initial point Q_1.

To estimate the influence variations of parameters of three-dimensional polar coordinate at the value dA_{12}, dD_{12} i dZ_{12} on the geodetic coordinates of point Q_2 its necessary to set functional connection between these values and proper variations of geodesic coordinates which are determined corrections $\Delta B''$ and $\Delta L''$. For this as an initial data used such formulas [2]:

$$dD_{12} = \left(M_2 + H_2\right)$$
$$\left(\cos A_{12} \sin Z_{12} \, a_1 - \sin A_{12} \sin Z_{12} \, a_2 + \cos Z_{12} \, a_3\right)$$
$$* dB_2 + \left(N_2 + H_2\right) \cos B_2 *$$
$$* \left(\cos A_{12} \sin Z_{12} \, b_1 + \sin A_{12} \sin Z_{12} \, b_2 -\right.$$
$$\left.- \cos Z_{12} \, b_3\right) dL_2 +$$
$$+ \left(\cos A_{12} \sin Z_{12} \, c_1 + \sin A_{12} \sin Z_{12} \, c_2\right.$$
$$\left.+ \cos Z_{12} \, c_3\right) dH_2,$$
$$D_{12} dZ_{12} = \left(M_2 + H_2\right)$$
$$\left(\cos A_{12} \cos Z_{12} \, a_1 - \sin A_{12} \cos Z_{12} \, a_2 - \sin Z_{12} \, a_3\right) * dB_2$$
$$+ + \left(N_2 + H_2\right) \cos B_2 *$$
$$* \left(\cos A_{12} \cos Z_{12} \, b_1 + \sin A_{12} \cos Z_{12} \, b_2 + \sin Z_{12} \, b_3\right) dL_2 +$$
$$+ \left(\cos A_{12} \cos Z_{12} \, c_1 + \sin A_{12} \cos Z_{12} \, c_2\right.$$
$$\left.- \sin Z_{12} \, c_3\right) dH_2,$$
$$D_{12} \sin A_{12} dA_{12} = -\left(M_2 + H_2\right)$$
$$\left(\sin A_{12} \, a_1 + \cos A_{12} \, a_2\right) dB_2 + \left(N_2 + H_2\right) \cos B_2 *$$
$$\left(- \sin A_{12} \, b_1 + \cos A_{12} \, b_2\right) dL_{2+}$$
$$\left(- \sin A_{12} \, c_1 + \cos A_{12} \, c_2\right) dH_2. \qquad (5)$$

In (5) the coefficients *a,b* and *c* can be determined from such expressions:

$$a_1 = \cos B_1 \cos B_2 + \sin B_1 \sin B_2 \cos l,$$
$$a_2 = \sin B_2 \sin l,$$
$$a_3 = \sin B_1 \cos B_2 - \cos B_1 \sin B_2 \cos l,$$
$$b_1 = \sin B_1 \sin l,$$
$$b_2 = \cos l,$$
$$b_3 = \cos B_1 \sin l,$$
$$c_1 = \cos B_1 \sin B_2 - \sin B_1 \cos B_2 \cos l,$$
$$c_2 = \cos B_2 \sin l,$$
$$c_3 = \sin B_1 \sin B_2 + \cos B_1 \cos B_2 \cos l. \qquad (6)$$

Analysing the structure of formulas (5) we can see that the expressions in square handles are fully determined at the values of coordinates of the proper points of ellipsoid. It allows to calculated them in time, and in subsequent transformations to use them in as the coefficients

$$k_1 = \cos A_{12} \sin Z_{12} \, a_1 - \sin A_{12} \sin Z_{12} \, a$$
$$+ \cos Z_{12} \, a_3,$$
$$k_2 = \cos A_{12} \sin Z_{12} \, b_1 + \sin A_{12} \sin Z_{12} \, b_2$$
$$- \cos Z_{12} \, b_3,$$
$$k_3 = \cos A_{12} \sin Z_{12} \, c_1 + \sin A_{12} \sin Z_{12} \, c_2$$
$$+ \cos Z_{12} \, c_3,$$
$$k_4 = \cos A_{12} \cos Z_{12} \, a_1 - \sin A_{12} \cos Z_{12} \, a_2$$
$$- \sin Z_{12} \, a_3,$$
$$k_5 = \cos A_{12} \cos Z_{12} \, b_1 + \sin A_{12} \cos Z_{12} \, b_2 +$$
$$\sin Z_{12} \, b_3,$$
$$k_6 = \cos A_{12} \cos Z_{12} \, c_1 + \sin A_{12} \cos Z_{12} \, c_2$$
$$- \sin Z_{12} \, c_3,$$
$$k_7 = \sin A_{12} \, a_1 + \cos A_{12} \, a_2,$$
$$k_8 = - \sin A_{12} \, b_1 + \cos A_{12} \, b_2,$$
$$k_9 = - \sin A_{12} \, c_1 + \cos A_{12} \, c_2. \qquad (7)$$

Taking into account these denotations formulas (5) will write:

$$dD_{12} = \left(M_2 + H_2\right) k_1\, dB_2 + \left(N_2 + H_2\right) \cos B_2\, k_2\, dL_2 + k_3\, dH_2,$$
$$D_{12} dZ_{12} = \left(M_2 + H_2\right) k_4\, dB_2 + \left(N_2 + H_2\right) \cos B_2\, k_5\, dL_2 + k_6\, dH_2$$
$$D_{12} \sin A_{12} dA_{12} = -\left(M_2 + H_2\right) k_7\, dB_2 + \left(N_2 + H_2\right) \cos B_2\, k_8\, dL_2 + k_9\, dH_2., \qquad (8)$$

Solution of three differential equalizations (8), which have three unknown corrections dB_2, dL_2 i dH_2 allows to define corrections to the three-dimensional geodesic coordinates of point of Q_2 for influence of variation of three-dimensional topocentric polar coordinates in an initial point. Therefore find

$$\Delta B_2'' = \rho'' \frac{D_{12}}{\left(M_2 + H_2\right) p_1} \left(\frac{\Delta D_{12}}{D_{12}} + p_2 \frac{\Delta Z_{12}''}{\rho''} + p_3 \sin A_{12} \frac{\Delta A_{12}''}{\rho''}\right),$$

$$\Delta L_2'' = \rho'' \frac{D_{12}}{\left(N_2 + H_2\right)\left(k_5 k_9 - k_6 k_8\right)} \sec B_2 \left(k_9 \frac{\Delta Z_{12}''}{\rho''} - \frac{M_2 + H_2}{D_{12}} \left(k_4 k_9 - k_6 k_7\right) \frac{\Delta B_2''}{\rho''} - k_6 \sin A_{12} \frac{\Delta A_{12}''}{\rho''}\right),$$

$$\Delta H_2'' = \frac{1}{k_9} \left(\left(M_2 + H_2\right) k_7 \frac{\Delta B_2''}{\rho''} - \left(N_2 + H_2\right) \cos B_2\, k_8 \frac{\Delta B_2''}{\rho''} + D_{12} \sin A_{12} \frac{\Delta A_{12}''}{\rho''}\right). \qquad (9)$$

In (9) the coefficients p can be determined from such expressions:

$$p_1 = \left(k_1 + \frac{k_3 k_7}{k_9}\right) - \frac{k_4 k_9 + k_6 k_7}{k_5 k_9 - k_6 k_8} * \left(k_2 - \frac{k_3 k_8}{k_9}\right),$$

$$p_2 = k_9 \frac{k_2 - \frac{k_3 k_8}{k_9}}{k_5 k_9 - k_6 k_8},$$

$$p_3 = -k_6 \frac{k_2 - \frac{k_3 k_8}{k_9}}{k_5 k_9 - k_6 k_8} + \frac{k_3}{k_9}. \qquad (10)$$

The final value of geodetic coordinate of point Q_2 and geodetic height of this point are determined taking into account formulas (4) and (9), from such expressions

$$B_{2\,k} = B_2 + \Delta B' + \Delta B_2'',$$
$$L_{2\,k} = L_2 + \Delta L_2' + \Delta L_2'',$$
$$H_{2\,k} = H_2 + \Delta H_2''. \qquad (11)$$

Conclusions

On the basis of the got formulas it is possible to do such conclusions:

- three-dimensional geodesic coordinates of point, the coordinates of which are determined in relation to an initial point go through changes, if geodesic and three-dimensional polar coordinates changed in an initial point;
- corrections to the geodesic coordinates take into account as influence of change in the initial point of parameters which are used for determination of common survey computation on-the-spot of ellipsoid, so changes of three-dimensional polar coordinates in this point;
- the values of coefficients a_i,b_i, c_i,k_i i p_i can be calculated after the initial values of coordinates in the points of research which will not influence on exactness of determination of corrections.

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Differential formulas of the first genus for determination of common survey computation in three-dimensional coordinate system

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The theory of determination of corrections to three-dimensional geodesic coordinates of adjacent point in relation to the initial point of geodesic network by differential formulas is considered. It is set that the final values of corrections include in constituents, which take into account both the changes of geodesic coordinates of initial point of network and changes of topocentric polar coordinates in this initial point.