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THE A PRIORI ESTIMATION OF PRECISION THEODOLITE LINES WHEN YOU RUN ADDITIONAL LINEAR AND ANGULAR MEASUREMENTS

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Topographic and cadastral surveys are performed, as a rule, from the points of theodolite lines. Often there are situations when the density of the initial geodetic points is not sufficient to make the traverse required accuracy in compliance with the required normative documents [2, 3] the lengths of traverses. The output is typically found in the crowding of the network of initial points or creating a system of theodolite lines.

One of the solutions of this problem may be the execution of additional linearly-angular measurements at the points of turn. Let’s see the effect of such measurements on the accuracy of determination of the points of a traverse, if one performs extra dimensions using schemes «through point» and «at each point».

The a priori estimation of precision of the position of a point traverse when running additional linear and angular measurements using scheme «through one point».

Estimating a priori precision one usually proceed from the fact that the traverse is elongated, has approximately equal sides and is based on the initial points and direction angles. The determining parameter, whose value should result in a priori precision estimation is the mean squared error of the position of a point in the weak place of traverse after adjustment. It is assumed that $2m_P = m_w$, where m_w - the position error of the last point traverse. It is calculated by the formula [1, 5]

$$m_w^2 = m_Q^2 + m_L^2 \qquad (1)$$

where $m_Q = \frac{m_\beta}{\rho} L \sqrt{\frac{n+3}{12}}$ - the mean square transverse error of line, calculated subject to prior angular distribution of the closing error, $m_L = m_d\sqrt{n}$ - the mean squared longitudinal error of the line, m_d - the mean square error of measurement length of sides, n - the number of sides in the traverse, m_β - mean square error of measurement of angles, L – the length of traverse.

From formula (1) can be find L [4]

$$L = \frac{\rho}{m_\beta}\sqrt{\frac{12(4m_P^2 - nm_d^2)}{(n+3)}}. \qquad (2)$$

Let it in the traverse there measured additional angles through the point of traverse (i = 1, 3,...), formed by the directions on the following adjacent points (Fig. 1). Here we suppose, that in the main traverse the number of angles is odd, and thus providing measurement of the additional angle at the last point of the traverse.

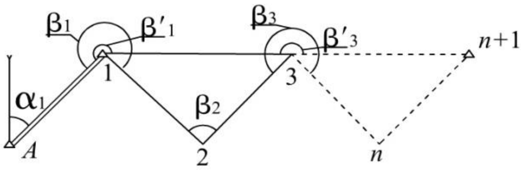

Fig. 1. The equilateral elongated theodolite traverse with extra dimensions corners «through one point»

Being measured in this manner angles give rise to condition of difference of directional angles.

$$\left(\alpha_1 + \sum_1^{n+1}\beta_i \pm 180^\circ\right) - \left(\alpha_1 + \sum_1^{n+1}\beta'_{2i-1} \pm 180^\circ\right) = 0.$$

Let’s accept for the error unit of weight the mean square error of angle measurement, i.e. $\mu = m_\beta$. Then the reverse weights q *of* measured angles will be equal to unity, $q = \frac{m_\beta^2}{\mu^2} = 1$ and matrix Q of measurement results will be the unity matrix. Let us find the inverse weight matrix $Q_{(\beta,\beta')}$ of adjusted values (β,β') by the formula $Q_{(\beta,\beta')} = E - QA^T(AQA^T)^{-1}A$ [6], transformed with considering that $Q = E$.

$$Q_{(\beta,\beta')} = E - A^T(AA^T)^{-1}A,$$

where A – the row matrix of coefficients of conditional equations of amendments, it will have the following structure $A = [1 \; \dots \; -1 \; -1 \; \dots]$. The number of columns in this matrix will be equal to the number of measured angles. After finding the inverse of the weight matrix $Q_{(\beta,\beta')}$ of adjusted values with dimensions $1{,}5n+2 \times 1{,}5n+2$, we will get the adjusted values of angular measurements

$$m_{\beta'} = m_\beta \sqrt{\frac{(1{,}5n+1)}{(1{,}5n+2)}}. \qquad (3)$$

It is obvious that increase of precision of the transverse shear increases in $\sqrt{\frac{(1{,}5n+2)}{(1{,}5n+1)}}$ times.

Suppose that in the equilateral elongated traverse sides, formed by three adjacent points, measured through one point (Fig. 2). Let's consider the first three points.

Assume that they are located on a straight line, then there is a condition of sides: $d_1 + d_2 - d_{1-3} = 0$.

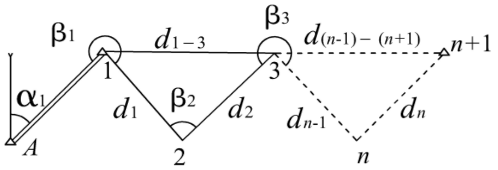

Fig. 2. The equilateral elongated theodolite traverse with extra dimensions of sides

Let's accept for the unit of weight error the mean squared error of distance, i.e. $\mu = m_d$. Then the reverse weights of the measured sides are equal to unity, $q = \frac{m_d^2}{\mu^2} = 1$. Inverse matrix Q of weights will be the unity matrix with the dimension 3×3.

The Row matrix of coefficients of conditional equations of amendments will be $A = [1 \quad 1 \quad -1]$.

Multiplying the matrix A of coefficients of conditional equations of amendments to its the transpose A^T, we obtain the matrix $(AA^T)^{-1}$ inverse of which is equal $\frac{1}{3}$. Now let's find the intersection $A^T (AA^T)^{-1} A = \frac{1}{3}$, and the difference $E - A^T (AA^T)^{-1} A = \frac{2}{3}$.

Thus, the mean square error of measuring the distance after adjustment will be

$$m_{d'} = \sqrt{\frac{2}{3}} m_d. \qquad (4)$$

One can see that the increase of precision of the longitudinal shear will be in $\sqrt{\frac{3}{2}}$ times.

Substituting adjusted quantities of angles and line lengths from the formulas (3) and (4) in the formula (1) and (2) we obtain

$$m_w = 2m_P = \sqrt{\frac{m_\beta^2}{\rho^2} L^2 \frac{1{,}5n+1}{1{,}5n+2} \frac{n+3}{12} + m_d^2 \frac{2}{3} n} \ ;$$

$$L = \frac{\rho}{m_\beta} \sqrt{\frac{1{,}5n+2}{1{,}5n+1}} \sqrt{\frac{12(4m_P^2 - \frac{2}{3} n m_d^2)}{(n+3)}}. \qquad (5)$$

A priori estimation of precision of the point position of **theodolite traverse when performing additional linear and angular measurements at each point of the traverse**

Let's suppose that angular measurements are made at each point of the stroke (Fig. 3), so they form a «chain of triangles».

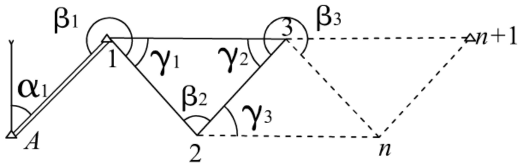

Fig. 3. Equilateral elongated theodolite traverse with extra dimensions angles on each point

In this case, the standard error of measurement of each angle on the point of the traverse after the adjustment for the condition of figures will be

$$m'_{\beta_i} = m'_{\gamma_i} = \sqrt{\frac{2}{3}} m_\beta. \qquad (6)$$

It is obvious that transverse shift in this case decreases in $\sqrt{\frac{3}{2}}$ times.

If the measurements of sides through the point of traverse were additionally made, it would lead to the emergence of n - 1 terms of sides. Matrix A of coefficients of conditional equations will have n - 1 rows and 2n - 1 columns. The researches have shown that the mean square error m'_d of linear measurements for the lengths participating in the measurements twice (all lengths are related to the running line, except the first and last) after the adjustment will be approximately equal

$$m'_d = 0{,}7 m_d \qquad (7)$$

Longitudinal shear will decrease approximately in $\frac{1}{0{,}7} = 1{,}4$ times.

Let's substitute the adjusted values of the angles and line lengths those are calculated by formulas (6) and (7) in the formula (1) and (2). As a result, we obtain

$$m_w = 2m_P = \sqrt{\frac{m_\beta^2}{\rho^2} L^2 \frac{n+3}{18} + 0{,}5 n m_d^2} \ ;$$

$$L = \frac{\rho}{m_\beta} \sqrt{\frac{18(4m_P^2 - 0{,}5 n m_d^2)}{(n+3)}}. \qquad (8)$$

Knowing m_P and entering m_β and m_d, by the formulas (2), (5) and (8) there can be calculated the allowable length L of traverses at a different number of sides n, or the allowable number of sides in the traverses of varying lengths. For example, to use electronic tacheometer for laying theodolite traverses with precision of measurement lengths $m_d = 5$ mm and angles $m_\beta = 7''$ to ensure the determining of the mean-square error of points positions $m_P = 0{,}05$ m (in numerator) and

$m_P = 0{,}10$ m (the denominator) possible lengths of traverses, and number of sides are presented in the tab.

Table

Length of traverses at varying number of sides

n	5	10	15	20
L, км по (2)	3,6/7,2	2,8/5,6	2,4/4,8	2,1/4,2
L, км по (5)	3,8/7,6	2,9/5,8	2,4/4,9	2,1/4,3
L, км по (8)	4,4/8,8	3,4/6,9	2,9/5,9	2,6/5,2

The research above demonstrates that one should resort to additional linear and angular measurements in theodolite traverses in exceptional cases, only if the number sides is up to 5, as it is not possible to achieve a significant increase in the stroke length.

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There have been obtained a formula for calculating the mean square error of the provisions of the endpoint of stroke and stroke length when measured more angles through the point of turn, formed by the directions on the following adjacent points progress vertex and the respective lengths of the parties, as well as when linearly-angular measurements are made at each point in the progress so as to form a «chain of triangles».