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COMPARATIVE ANALYSIS OF FORMULAS FOR THE POTENTIAL AND ITS RADIAL DERIVATIVES FOR THREE-LAYERED SPHERICAL AND ELLIPSOIDAL PLANETS

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Statement of the problem. To determine the contribution of ellipsoidal shape of the planet in the potential and its radial derivatives we need the formula to calculate it. To check the obtained value it necessary to compare them with the known expressions for spherical planet arising from classical potential theory.

An analysis of recent research and publications. Potential homogeneous spherical and ellipsoidal planet is determined by the known formulas [4, 8]. If the distribution of mass inside the planet - piecewise continuous function, then algorithm for determining became extremely complicated and for spherically symmetric case, it is given in [2, 3]. Those relationships, that determine the value of the gravitational field for the ellipsoid are in [3]. However, the formula for the radial derivatives are not given.

Statement of the task. To determine the potential and its radial derivatives up to the second order for spherical and ellipsoidal planet and to compare them with each other.

The main material. For terrestrial planets (Earth, Mars, Venus) and Moon the mass distribution is assumed by piecewise continuous function, and the number of jumps is determined equal to three, so the function of jumps looks

$$\delta=\begin{cases}\delta_1, & \rho\in G_1 \;\; 0=\rho_0\le\rho\le\rho_1 \;\;;\\ \delta_2, & \rho\in G_2 \;\; \rho_1\le\rho\le\rho_2 \;\;;\\ \delta_3, & \rho\in G_3 \;\; \rho_2\le\rho\le\rho_3=1 \;\;;\end{cases} \qquad (1)$$

where ρ_0, ρ_1, ρ_2, ρ_3 - - the relative radii of occurrence of jumps and points $P \;\; \rho,\theta,\lambda$.

The question of contribution of this distribution in internal and external potential of the planet is arised. We use the technique described in [3]. Consider it in more detail for this case.

Potential V of the point $P\in G_i$ of piecewise continuous mass distribution in the bowels of the planet is the sum of internal potential W_i^b in G_i, external W_i^z for bodies G_i (i < l), and permanent W_i^p in cavities of spherical segments G_i (i > l):

$$W_i^z = f\sum_{i=1}^{l-1}\frac{M_{G_i}}{\rho\cdot R}\,,\;\; M_{G_i}=\delta_i \;\; \rho_i^3-\rho_{i-1}^3 \;\; V_k\,,$$

$$W_i^b=\frac{fV_k\delta_i}{2R}\;\; 3\rho_i^2-\rho^2 \;\;,$$

$$W_i^p=\frac{3fV_k\delta_i}{2R}\;\; \rho_i^2-\rho_{i-1}^2 \;\;, \qquad (2)$$

where V_k - volume of the sphere (for ellipsoid - V_τ), f - gravitational constant, R - radius of the sphere.

For ellipsoidal bodies τ_i (for simplicity, we consider an ellipsoid of rotation) also it is distinguished three types of potentials: internal - $W_{\tau_i}^b \;\; \delta_j$, external - $W_{\tau_i}^z \;\; \delta_j$, permanent - $W_{G_i}^P \;\; \delta_j$ - in cavity of ellipsoidal segment $G_i \;\; \rho_{i-1}\le\rho\le\rho_i$:

$$W_{\tau_i}^b \;\; \delta_j \;\; =\frac{3fV_\tau}{4}\delta_j\Big[\rho_i^2M_{00} \;\; 0 \;\; -,$$

$$-\rho^2\Big[M_{10} \;\; 0 \;\; \sin^2\theta+M_{01} \;\; 0 \;\; \cos^2\theta\Big]\Big],\quad P\in\tau_i\,,$$

$$W_{\tau_i}^z \;\; \delta_j \;\; =\frac{3}{4}fV_\tau\delta_j\rho_i^3\left[M_{00}\left(\frac{\xi}{\rho_i^2}\right)-\right.$$

$$\left.-\rho^2\left[M_{10}\left(\frac{\xi}{\rho_i^2}\right)\sin^2\theta+M_{01}\left(\frac{\xi}{\rho_i^2}\right)\right]\right],\quad P\notin\tau_i\,, \qquad (3)$$

$$W_{G_i}^P \;\; \delta_j \;\; =\frac{3fV_\tau}{4}\delta_j \;\; \rho_i^2-\rho_{i-1}^2 \;\;,\quad P\notin G_i\cap P\in\tau_{i-1}\,.$$

For calculating of value $M_{kt} \;\; \xi$ we determine root of the equation $\frac{x^2+y^2}{a^2+\xi}+\frac{z^2}{c^2+\xi}=1$, the solution of which in spherical coordinates is:

$$\frac{\xi}{a^2}=\frac{\rho^2 \;\; \sin^2\theta+\gamma^2\cos\theta \;\; - \;\; 1+\gamma^2}{2}+$$

$$+\frac{\sqrt{\rho^2 \;\; \sin^2\theta+\gamma^2\cos\theta \;\; - \;\; 1+\gamma^2 \;\; {}^2-4\gamma^2 \;\; 1-\rho^2}}{2}$$

,

$$\gamma=\frac{c}{a}\,. \qquad (4)$$

It is important to note, that the ellipsoidal coordinate $\frac{\xi}{a_i^2}$ on the ellipsoid τ_i $(0 \le \rho \le \rho_i)$ is expressed relative to the fundamental τ as following:

$$\frac{\xi}{a_i^2}=\frac{\xi}{a^2}\left(\frac{1}{\rho_i}\right)^2. \qquad (5)$$

Then values $M_{kt}(\xi)$ can be written as

$$M_{kt}(\xi)=a^{2k}c^{2t}\int_{0,\xi}^{\infty}\frac{du}{(a^2+u)^{k+1}(c^2+u)^{t+\frac{1}{2}}}=$$

$$=\gamma^{2t}\int_{0,\xi}^{\infty}\frac{du}{\left(1+\frac{u}{a^2}\right)^{k+1}\left(\gamma^2+\frac{u}{a^2}\right)^{t+\frac{1}{2}}a^3}=$$

$$=2\gamma^{2t}\int_{\sqrt{\gamma^2+\frac{\xi}{a^2}}}^{\infty}\frac{dz}{z^{2t+1}(z^2+e^2)^{k+1}a}. \qquad (6)$$

The value of (6) can be found through the use of recurrent relations [6], but it is more appropriate to calculate it using expansions in series. Then we get the following:

$$M_{kt}(\xi)=\frac{2\gamma^{2t}}{(2k+2t+1)\,\xi^{\frac{2k+2t+1}{2}}}+$$

$$+\frac{2\gamma^{2t}}{k!}\sum_{l=1}^{\infty}\frac{(1-\gamma^2)^{2l}(k+l)!}{l!\,(2k+2t+2l+1)\left(\gamma^2+\frac{\xi}{a^2}\right)^{\frac{2k+2t+2l+1}{1}}}.$$

Let write potentials of spherical planet for various cases:

1. $P\in G_1=\tau_0$,

$$V_1=W^b_{G_1}+W^p_{G_2}+W^p_{G_3}=$$

$$=\frac{fV_k}{2R}\left[\delta_1(3\rho_1^2-\rho^2)+3\sum_{i=2}^{3}\delta_i(\rho_i^2-\rho_{i-1}^2)\right],$$

2. $P\in G_2$, $V_2=W^z_{G_1}+W^b_{G_2}+W^p_{G_3}$, where

$$W^z_{G_1}=\frac{fM_{G_1}}{\rho R}=\frac{fV_k}{R}\frac{\delta_1\rho_1^3}{\rho},$$

$$W^p_{G_3}=\frac{3fV_k\delta_3}{2R}(\rho_3^2-\rho_2^2),$$

$$W^b_{G_2}=W^b_{\tau_2}-W^z_{\tau_1}=\frac{fV_k}{2R}\left[(3\rho_2^2-\rho^2)\delta_2-\frac{\delta_2\rho_1^3}{\rho}\right].$$

3. $P\in G_3$, $V_3=W^z_{G_1}+W^z_{G_2}+W^b_{G_3}$, where

$$W^z_{G_1}=\frac{fV_k}{R}\frac{\delta_1\rho_1^3}{\rho},\quad W^z_{G_2}=\frac{fV_k}{\rho R}(\rho_2^3-\rho_1^3)\delta_2,$$

$$W^b_{G_3}=W^b_{\tau_3}-W^z_{\tau_2}=\frac{fV_k}{2R}\left[(3\rho_3^2-\rho^2)\delta_3-\frac{2\delta_3\rho_2^3}{\rho}\right].$$

4. $P\notin\tau$,

$$V_4=\frac{fV_k}{R\rho}\left[\delta_1\rho_1^3+\delta_2(\rho_2^3-\rho_1^3)+\delta_3(\rho_3^3-\rho_2^3)\right].$$

Thus, the potential is defined as

$$V=\begin{cases}\frac{1}{2}\frac{fV_k}{R}\left[\delta_1(3\rho_1^2-\rho^2)+3\sum_{i=2}^{3}\delta_i(\rho_i^2-\rho_{i-1}^2)\right], \\ \qquad P\in G_1\ (0=\rho_0\le\rho\le\rho_1); \\ \frac{fV_k}{R}\left[\frac{(\delta_1-\delta_2)\rho_1^3}{\rho}+\frac{\delta_2(3\rho_2^2-\rho^2)}{2}+\frac{3\delta_3(\rho_3^2-\rho_2^2)}{2}\right], \\ \qquad P\in G_2\ (\rho_1\le\rho<\rho_2); \\ \frac{fV_k}{R}\left[\frac{(\delta_1-\delta_2)\rho_1^3}{\rho}+\frac{(\delta_2-\delta_3)\rho_2^3}{\rho}+\frac{\delta_3}{2}(3\rho_3^2-\rho^2)\right], \\ \qquad P\in G_3\ (\rho_2<\rho\le\rho_2=1); \\ \frac{fV_k}{R}\left[\frac{\delta_1\rho_1^3+\delta_2(\rho_2^3-\rho_1^3)+\delta_3(\rho_3^3-\rho_2^3)}{\rho}\right], \\ \qquad P\in G_3\ (\rho_2<\rho\le\rho_2=1)\end{cases} \qquad (7)$$

Then, its first and second derivatives are calculated, respectively:

$$\frac{\partial V}{\partial\rho}=\begin{cases}-\frac{fV_k\delta_1}{R}\rho, & P\in G_1; \\ -\frac{fV_k}{R}\left[\frac{(\delta_1-\delta_2)\rho_1^2}{\rho^2}+\delta_2\rho\right], & P\in G_2; \\ -\frac{fV_k}{\rho^2R}\left[(\delta_1-\delta_2)\rho_1^2+\rho_2^2(\delta_2-\delta_3)+\delta_3\rho^3\right], & \\ & P\in G_3; \\ -\frac{fV_k}{\rho^2R}\left[\delta_1\rho_1^3+\delta_2(\rho_2^3-\rho_1^3)+\delta_3(\rho_3^3-\rho_2^3)\right], & \\ & P\in R^3\setminus\tau;\end{cases} \qquad (8)$$

$$\frac{\partial^2V}{\partial\rho^2}=\begin{cases}-\frac{fV_k\delta_1}{R}, & P\in G_1; \\ \frac{fV_k}{R}\left[\frac{2(\delta_1-\delta_2)\rho_1^3}{\rho^3}-\delta_2\right], & P\in G_2; \\ \frac{fV_k}{R}\left[\frac{2\left((\delta_1-\delta_2)\rho_1^2+\rho_2^2(\delta_2-\delta_3)\right)}{\rho^3}-\delta_3\right], & \\ & P\in G_3; \\ \frac{fV_k}{R}\left[\frac{2\left(\delta_1\rho_1^3+\delta_2(\rho_2^3-\rho_1^3)\right)}{\rho^3}+\delta_3(\rho_3^3-\rho_2^3)\right] & \\ & P\in R^3\setminus\tau.\end{cases} \qquad (9)$$

Similarly, we define the potential and its derivatives for piecewise continuous function of the mass distribution of the ellipsoidal planet τ :

1. $V = W^b_{G_1}(\delta_1) + W^P_{G_2}(\delta_1) + W^P_{G_3}(\delta_1)$, $P \in G_1$.

(10)

2. $$V = W^z_{G_1}(\delta_1) + W^b_{G_2}(\delta_2) + W^P_{G_3}(\delta_3) = \\ = W^z_{\tau_0}(\delta_1 - \delta_1) + W^b_{\tau_1}(\delta_2) + \\ + W^P_{G_3}(\delta_3) , \quad P \in G_2 . \qquad (11)$$

3. $$V = W^z_{G_1}(\delta_1) + W^z_{G_2}(\delta_2) + W^b_{G_3}(\delta_3) = \\ = W^z_{\tau_0}(\delta_1 - \delta_2) + W^z_{\tau_1}(\delta_2 - \delta_2) + \\ + W^b_{\tau}(\delta_3) , P \in G_3 . \qquad (12)$$

4. $$V = W^z_{G_1}(\delta_1) + W^z_{G_2}(\delta_2) + W^z_{G_3}(\delta_3) = \\ = W^z_{\tau_0}(\delta_1 - \delta_2) + W^z_{\tau_1}(\delta_2 - \delta_3) + \\ + W^z_{\tau}(\delta_3) , \quad P \notin \tau . \qquad (13)$$

We write formulas, given above (10) - (13), in expanded form:

$$V = \frac{3}{2} f V_\tau \begin{cases} \frac{\delta_1}{2}\left(\rho_1^2 M_{00}(0) - \rho^2\left[M_{10}(0)\sin^2\theta + M_{01}(0)\cos^2\theta\right]\right) + \sum_{i=2}^{3}\delta_i\left(\rho_i^2 - \rho_{i-1}^2\right), & P \in G_1; \\ \frac{(\delta_1 - \delta_2)\rho_1^3}{2}\left(M_{00}\left(\frac{\xi}{\rho_1^2}\right) - \rho^2\left[M_{10}\left(\frac{\xi}{\rho_1^2}\right)\sin^2\theta + M_{01}\left(\frac{\xi}{\rho_1^2}\right)\cos^2\theta\right]\right) + \\ + \frac{\delta_2}{2}\left(\rho_1^2 M_{00}(0) - \rho^2\left[M_{10}(0)\sin^2\theta + M_{01}(0)\cos^2\theta\right]\right) + \delta_3\left(\rho_3^2 - \rho_2^2\right), & P \in G_2; \\ \sum_{i=1}^{2}\frac{(\delta_i - \delta_{i+1})\rho_i^3}{2}\left(M_{00}\left(\frac{\xi}{\rho_i^2}\right) - \rho^2\left[M_{10}\left(\frac{\xi}{\rho_i^2}\right)\sin^2\theta + M_{01}\left(\frac{\xi}{\rho_i^2}\right)\cos^2\theta\right]\right) + \\ \qquad + \frac{\delta_3}{2}\left(\rho_3^2 M_{00}(0) - \rho^2\left[M_{10}(0)\sin^2\theta + M_{01}(0)\cos^2\theta\right]\right), & P \in G_3; \\ \sum_{i=1}^{2}\frac{(\delta_i - \delta_{i+1})\rho_i^3}{2}\left(M_{00}\left(\frac{\xi}{\rho_i^2}\right) - \rho^2\left[M_{10}\left(\frac{\xi}{\rho_i^2}\right)\sin^2\theta + M_{01}\left(\frac{\xi}{\rho_i^2}\right)\cos^2\theta\right]\right) + \\ \qquad + \frac{\delta_3}{2}\left(\rho_3^2 M_{00}(\xi) - \rho^2\left[M_{10}(\xi)\sin^2\theta + M_{01}(\xi)\cos^2\theta\right]\right), & P \notin \tau. \end{cases} \qquad (14)$$

Differentiation of (14) includes a variable integration using the following equation:

$$\frac{\partial}{\partial x}\int_{\varphi(x)}^{\infty} p(x,u)\frac{du}{Q(u)} = \int_{\varphi(x)}^{\infty}\frac{\partial}{\partial x}p(x,u)\frac{du}{Q(u)} - \\ - p(x,\varphi(x))\frac{1}{Q(\varphi(x))}\frac{\partial}{\partial x}\varphi(x) . \qquad (15)$$

It should be noted that in finding first derivative the lower limit can be considered as constant, since $\varphi(x)$ satisfies the equation

$$p(x,\varphi(x)) = \\ = \frac{x^2}{a^2 + \varphi(x)} + \frac{y^2}{b^2 + \varphi(x)} + \frac{z^2}{c^2 + \varphi(x)} - 1 = 0 . \qquad (16)$$

Therefore, for the radial derivative it can be written

$$\frac{\partial V}{\partial \rho} = \frac{3}{2} f V_\tau \rho \begin{cases} -\delta_1\left[M_{10}(0)\sin^2\theta + M_{01}(0)\cos^2\theta\right], & P \in G_1; \\ -\rho_1^3(\delta_1 - \delta_2)\left[M_{10}\left(\frac{\xi}{\rho_1^2}\right)\sin^2\theta + M_{01}\left(\frac{\xi}{\rho_1^2}\right)\cos^2\theta\right] - \delta_2\left[M_{10}(0)\sin^2\theta + M_{01}(0)\cos^2\theta\right], & P \in G_2; \\ -\sum_{i=1}^{2}(\delta_i - \delta_{i+1})\rho_i^3\left[M_{10}\left(\frac{\xi}{\rho_i^2}\right)\sin^2\theta + M_{01}\left(\frac{\xi}{\rho_i^2}\right)\cos^2\theta\right] - \\ \qquad - \delta_3\left[M_{10}(0)\sin^2\theta + M_{01}(0)\cos^2\theta\right], & P \in G_3; \\ -\sum_{i=1}^{2}(\delta_i - \delta_{i+1})\rho_i^3\left[M_{10}\left(\frac{\xi}{\rho_i^2}\right)\sin^2\theta + M_{01}\left(\frac{\xi}{\rho_i^2}\right)\cos^2\theta\right] - \\ \qquad - \delta_3\left[M_{10}(\xi)\sin^2\theta + M_{01}(\xi)\cos^2\theta\right], & P \notin \tau. \end{cases} \qquad (17)$$

Finding the second derivative is more complicated,because the formula (15) is used fully.

So,

$$\frac{\partial^2 V}{\partial\rho^2}=\frac{3}{2}fV_\tau\begin{cases}-\delta_1\left[M_{10}(0)\sin^2\theta+M_{01}(0)\cos^2\theta\right], & P\in G_1;\\ -\rho_1^3(\delta_1-\delta_2)\left[M_{10}\left(\frac{\xi}{\rho_1^2}\right)\sin^2\theta+M_{01}\left(\frac{\xi}{\rho_1^2}\right)\cos^2\theta\right]- & \\ \quad-\delta_2\left[M_{10}(0)\sin^2\theta+M_{01}(0)\cos^2\theta\right]- & \\ \quad-\rho_1^3(\delta_1-\delta_2)\frac{\partial\xi}{\partial\rho}\frac{\rho}{\left(1+\frac{\xi}{a^2}\right)\left(\gamma^2+\frac{\xi}{a^2}\right)}, & P\in G_2;\\ -\sum_{i=1}^{2}(\delta_i-\delta_{i+1})\rho_i^3\left[M_{10}\left(\frac{\xi}{\rho_i^2}\right)\sin^2\theta+M_{01}\left(\frac{\xi}{\rho_i^2}\right)\cos^2\theta\right]- & \\ \quad-\sum_{i=1}^{2}(\delta_i-\delta_{i+1})\rho_i^3\frac{\partial\xi}{\partial\rho}\frac{\rho}{\left(1+\frac{\xi}{a^2}\right)\left(\gamma^2+\frac{\xi}{a^2}\right)}- & \\ \quad-\delta_3\left[M_{10}(0)\sin^2\theta+M_{01}(0)\cos^2\theta\right], & P\in G_3;\\ -\sum_{i=1}^{2}(\delta_i-\delta_{i+1})\rho_i^3\left[M_{10}\left(\frac{\xi}{\rho_i^2}\right)\sin^2\theta+M_{01}\left(\frac{\xi}{\rho_i^2}\right)\cos^2\theta\right]- & \\ \quad-\sum_{i=1}^{2}(\delta_i-\delta_{i+1})\rho_i^3\frac{\partial\xi}{\partial\rho}\frac{\rho}{\left(1+\frac{\xi}{a^2}\right)\left(\gamma^2+\frac{\xi}{a^2}\right)}- & \\ \quad-\delta_3\left[M_{10}(\xi)\sin^2\theta+M_{01}(\xi)\cos^2\theta\right]- & \\ \quad-\frac{\partial\xi}{\partial\rho}\frac{\delta_3\rho}{\left(1+\frac{\xi}{a^2}\right)\left(\gamma^2+\frac{\xi}{a^2}\right)}, & P\notin\tau.\end{cases} \tag{18}$$

In this case the value $\frac{\partial\xi}{\partial\rho}$ is defined as follows:

$$\frac{\partial\xi}{\partial\rho}=a^2\rho\left(\sin^2\theta+\gamma^2\cos\theta\right)\times\left(1+\frac{\rho^2\left(\sin^2\theta+\gamma^2\cos\theta\right)-\left(1+\gamma^2\right)+2\gamma^2}{\sqrt{\left(\rho^2\left(\sin^2\theta+\gamma^2\cos\theta\right)-\left(1+\gamma^2\right)\right)^2-4\gamma^2\left(1-\rho^2\right)}}\right). \tag{19}$$

Parameters of three-layer model of the density we will obtain for the Earth, averaging known reference model PREM [2], and the parameters of the ellipsoid from [4]. The results are recorded in the Table 1.

Table 1

Basic parameters of three-layered planet Earth

$fM\cdot10^9$ $м^2/c^2$	$a_c\cdot10^3$ $м$	Average density δ_c $г\cdot см^{-3}$	Average radius $R\cdot10^3 м$	Core radius $R_я\cdot10^3 м$	Radius of the mantle core $R_м\cdot10^3 м$	α	Density $г\cdot см^{-3}$		
							in the core	in the mantle	in the crust
398603	6378	5,514	6371	3480	6346,6	1:298,256	10,998	4,4754	2,52

Graphs of potential and its first two radial derivatives for the above planets are represented in Figure 1 and the results of calculations by formulas (7) - (9) and (14), (17) and (18) coincide if $a=b=c=R$.

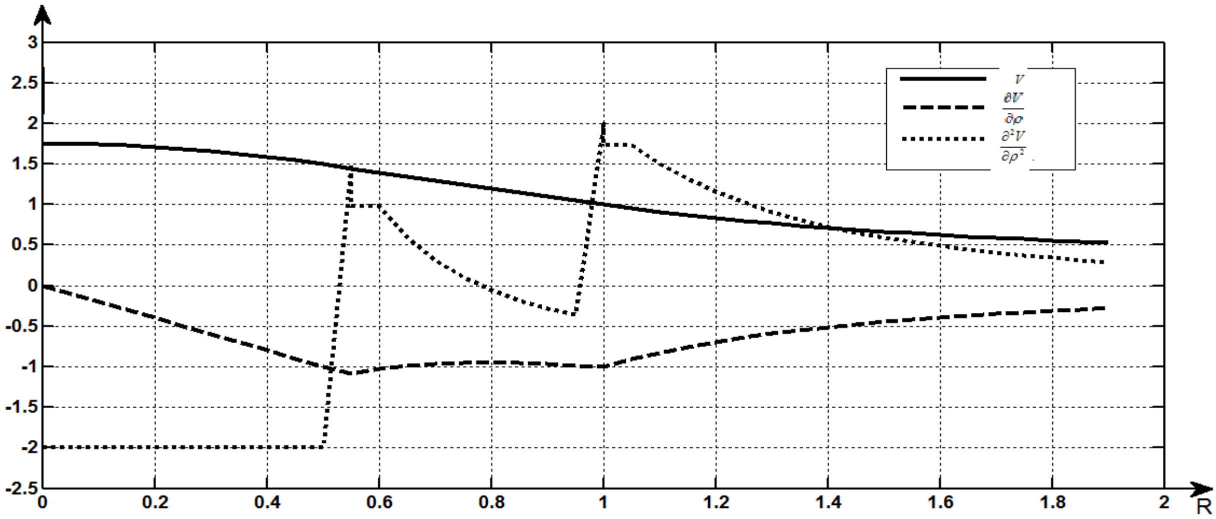

Fig.1. Graphs of potential and its first two radial derivatives for the Earth

Using formulas (14), (17), (18), lets calculate the values of the potential and its derivatives for the ellipsoid and the results will be provided in Table 2.

Table 2

The potential difference of ellipsoidal and spherical Earth

ρ	$V \cdot 0{,}625652 \cdot 10^8$ $м^2/c^2$	$\Delta V \cdot 0{,}625652 \cdot 10^8$ $м^2/c^2$					
		0°	30°	60°	90°	120°	150°
0	1.742804	0.003016	0.003016	0.003016	0.003016	0.003016	0.003016
0.1	1.732831	0.002915	0.002915	0.002915	0.002915	0.002915	0.002915
0.2	1.702912	0.002611	0.002611	0.002611	0.002610	0.002611	0.002611
0.3	1.653048	0.002105	0.002104	0.002103	0.002103	0.002103	0.002104
0.4	1.583239	0.001396	0.001395	0.001393	0.001392	0.001393	0.001395
0.5	1.493484	0.000485	0.000483	0.000480	0.000478	0.000480	0.000483
0.6	1.388607	0.002596	0.002595	0.002592	0.002590	0.002592	0.002595
0.7	1.289950	0.002060	0.002059	0.002057	0.002056	0.002057	0.002059
0.8	1.194651	0.001442	0.001441	0.001439	0.001437	0.001439	0.001441
0.9	1.098886	0.000742	0.000741	0.000737	0.000735	0.000737	0.000741
1	1.000367	0.000009	0.000011	0.000015	0.000018	0.000015	0.000011
1.1	0.909425	0.000010	0.000010	0.000011	0.000012	0.000011	0.000010
1.2	0.833639	0.000010	0.000009	0.000008	0.000008	0.000008	0.000009
1.3	0.769513	0.000010	0.000008	0.000006	0.000006	0.000006	0.000008
1.4	0.714548	0.000009	0.000007	0.000005	0.000004	0.000005	0.000007
1.5	0.666912	0.000009	0.000007	0.000004	0.000003	0.000004	0.000007

Conclusions. Numerical results obtained on the basis of described above formulas which illustrated graphically, are proving the properties of found potentials and their derivatives: continuity of V and its radial derivatives; gap of at depths of changing the values of the second derivative, when moving through the body surface. Calculations by formulas for ellipsoid coincide with those made for the sphere. The difference between the values of potentials in spherical and ellipsoidal case, which is given in Table 2, confirms the need to consider the ellipticity in the relevant studies.

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Comparative analysis of formulas for the potential and its radial derivatives for three-layered spherical and ellipsoidal planets

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Formulas are derived for the potential of ellipsoid and its radial derivatives up to the second order. Their comparison with the known relations for spherical planet have been done.