ANALYSIS OF WAYS TO ESTIMATE ACCURACY OF A PLOT AREA MEASUREMENT BY ANALYTICAL METHOD

Ye.I. Smirnov

Lviv National Agrarian University

Key words: area, plot, accuracy, coordinates.

Problem setting

There are new requirements to accuracy of measurement of a plot boundaries, in particular: increase of accuracy of measurement of a plot area; correspondence of area, recorded in state acts on right of ownership of a plot to actual area; grounding of the record accuracy of area in documents determining the right on land.

For the beginning it is necessary to define marks, which are going to be used. In most of geodesic literature area is marked by letter P , in the rest of sources it is marked by letter S , and letter P marks perimeter, current points are marked by letter i.

Thus, mean square error of area measurement and coordinates of a plot's apexes are marked by letters *ms* and *mi* respectively. Main requirements to topographic-geodesic data, stated by "Principal regulations as to formation of topographic-geodesic base" and "Regulations of land and cadaster inventory of settled land" [11] are described in the table 1.

Still one should note, that those characteristics do not absolutely agree [1-12].

Analysis of the latest researches and publications

To define error of area measurement of land plots in work [1, 6, 7] the following formula is used

$$m_S = m_i \sqrt{S} \sqrt{\frac{1+k^2}{2k}}, \qquad (1)$$

where m_i – is an error of position of apex; S – is a plot area; k – is a coefficient of elongation (relation of a plot's length to its width).

Table 1.

Calculation of accuracy of area measurement of agricultural enterprises and organizations

Specialization of a farm (2-4 people)	Average characteristics		Mean errors	
	area (ha)	size (m×m)	average square (m^2)	critical (ha)
1.Production of green crops	0,05	20x25 10x50	1 2	0,0006
2. Production of vegetables	7,00	250x280 100x700	15 28	0,0084
3. Production of sugar beet	15,00	375x400 200x750	22 31	0,0930
4. Production of flax	20,00	400x500 200x1000	26 41	0,0123
5. Production of cereals and cattle-breeding	150,00	1200x1250 400x3750	69 150	0,0450

Let us transform the formula for determination of m.s.e. of coordinates of boundary marks basing on defined accuracy of record unit

$$m_i = \frac{m_S}{\sqrt{S}\sqrt{\frac{1+k^2}{2k}}}. \qquad (2)$$

There is another formula [3, 9, 10, 14] to determine the error

$$m_S = m_i \sqrt{S} + KS, \qquad (3)$$

where K – is a coefficient that depends on size, shape, accuracy and number of points on the outline of a land plot, which can be determined according to the following formula

$$K = \frac{mq\left(\sqrt{\frac{1+k^2}{2k}} - 1\right)}{\sqrt{S}}, \qquad (4)$$

where $q = \sqrt{2n + \frac{1}{n+1}}$ - is an indicator of a spacing of outline; n – is a number of intermediate points on a plot outline; $k = a/b$ – is a coefficient of elongation of a rectangular land plot.

Or

$$m_S = \frac{m_i\sqrt{S}}{3}\left(1+\sqrt{S}\sqrt{2n+\frac{1}{n+1}}\left(\sqrt{\frac{1+k^2}{2k}}-1\right)\right), \quad (5)$$

m.s.e. of determination of coordinates of boundary marks can be defined according to the formula

$$m_i = \frac{3m_S}{\sqrt{S}\left(1+\sqrt{2n+\frac{1}{n+1}}\left(\sqrt{\frac{1+k^2}{2k}}-1\right)\right)} \quad (6)$$

In the work [9] in the formula (4) coefficient q looks a bit differently

$$q = \frac{\sqrt{2n+1}}{(n+1)}.$$

Taking into account the fact, accuracy of area measurement is determined according to the following formula

$$m_S = \frac{m_i}{3}\sqrt{S}\left(1+\frac{\sqrt{2n+1}}{(n+1)}\sqrt{\frac{1+k^2}{2k}}-1\right) \quad (7)$$

To determine m.s.e. of coordinates of boundary marks one applies the following formula:

$$m_i = \frac{3m_S}{\sqrt{S}\left(1+\sqrt{2n+\frac{1}{n+1}}\left(\sqrt{\frac{1+k^2}{2k}}-1\right)\right)}. \quad (8)$$

In the work [4] we have got another formula, basing on the one proposed by A.Maslov (1)

$$m_S = \frac{m_i}{2}\Sigma\sqrt{(x_{i+1}-x_{i-1})^2+(y_{i+1}-y_{i-1})^2} = \frac{m_i}{2\sqrt{2}}\sqrt{[DD}_{i-1}^{i+1}, \quad (9)$$

where $[DD]_{i-1}^{i+1}$ is an amount of diagonals, connecting apexes $i+1$ i $i-1$.

$$m_i = \frac{2m_S\sqrt{2}}{\sqrt{[DD]_{i-1}^{i+1}}}. \quad (10)$$

Besides the mentioned works, there are two more [8] which have applied the formula

$$S = \frac{m_S^2}{m_i^2 \sin\frac{360^o}{n}}, \quad (11)$$

(3)

thus

$$m_S = m_i\sqrt{S\sin\frac{360^o}{n}}. \quad (12)$$

Analysis of the formula shows, that accuracy of measurement of area does not depend on the shape of land plot, but it is not always truth.

The next work [12] applies the formula

$$m_S = m_i\sqrt{S}\sqrt{\mathrm{ctg}\frac{180^o}{n}}. \quad (13)$$

Task setting

Considering the fact that the authors use rectangular land plots, we can determine mean-square errors of measurement of area of land plots according to the formulas (1) and (3), the received calculations are presented in the table 2-7.

Table 2.

M.s.e. of measurement of area of land plots for specialization of a farm according to the formula (1)

m_S (m^2)							Special. of farm
k	1	2	4	5	7	9	
	0,2236	0,1936	0,1768	0,1732	0,1690	0,1667	1
	6,1734	5,3463	4,8805	4,7819	4,6667	4,6014	2
	9,0370	7,8262	7,1443	7,0000	6,8313	6,7358	3
	19,379	16,783	15,321	15,011	14,649	14,444	4
	163,30	141,42	129,10	126,49	123,44	121,716	5

Table 3.

M.s.e. of measurement of area of land plots for specialization of a farm according to the formula (5)

m_S (m^2)							Special. of farm
k	1	2	4	5	7	9	
	0,2236	0,5438	0,9028	1,0335	1,2504	1,4310	1
	6,173	15,012	24,924	28,534	34,521	39,507	2
	9,037	21,977	36,487	41,770	50,534	57,833	3
	19,379	47,126	78,239	89,574	108,37	124,019	4
	163,30	397,11	659,28	754,79	913,16	1045,05	5

Having analyzed results of the calculations, we make a conclusion that the results completely coincide only in case of square-shaped plots, and at $k = 9$ discrepancy increases 8 times (121,716 and 1045, 05) for production of cereals and cattle-breeding according to the formulas (1) and (5).

We have made diagrams, demonstrating changes of land area

Table 4.

M.s.e. of measurement of area of land plots for specialization of a farm according to the formula (7)

m_S (m^2)							Special. of farm
k	1	2	4	5	7	9	
	0,2236	0,2736	0,3297	0,3501	0,3840	0,4122	1
	6,173	7,554	9,102	9,666	10,601	11,379	2
	9,037	11,058	13,324	14,149	15,518	16,658	3
	19,379	23,713	28,572	30,342	33,277	35,721	4
	163,30	199,81	240,77	255,68	280,41	301,005	5

Table 5.

M.s.e. of measurement of area of land plots for specialization of a farm according to the formula (9)

m_S (m^2)							Special. of farm
k	1	2	4	5	7	9	
	0,671	0,750	0,978	1,082	1,2677	1,4318	1
	18,520	20,706	26,998	29,863	35,000	39,529	2
	27,111	30,311	39,521	43,715	51,235	57,8658	3
	58,14	65,00	84,75	93,74	109,87	124,108	4
	489,90	547,72	714,14	789,94	925,82	1045,63	5

Table 6.

M.s.e. of measurement of area of land plots for specialization of a farm according to the formula (12)

Special. of farm	1	2	3	4	5
m_S	0,634	17,511	25,634	54,970	463,205

Table 7.

M.s.e. of measurement of area of land plots for specialization of a farm according to the formula (13)

Special. f farm	1	2	3	4	5
m_S	0,527	14,552	21,302	45,681	384,927

Having analyzed results of the calculations, we make a conclusion that the results completely coincide only in case of square-shaped plots, and at $k = 9$ discrepancy increases 8 times (121,716 and 1045, 05) for production of cereals and cattle-breeding according to the formulas (1) and (5).

We have made diagrams, demonstrating changes of land area

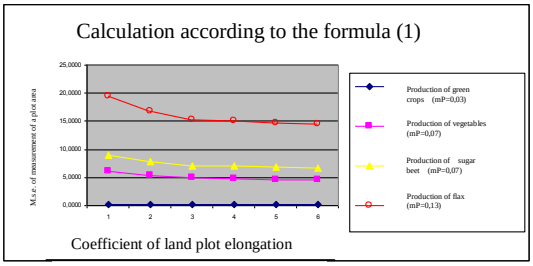

Fig. 1. Diagram of errors of measurement of land plots area according to the formula (1) for specialization of a farm producing green crops, vegetables, sugar beet, flax, cereals and cattle-breeding

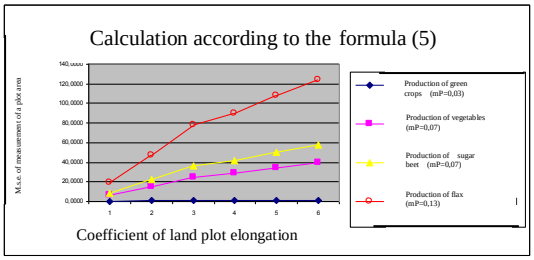

Fig. 2. Diagram of errors of measurement of land plots area made according to the formula (5) for specialization of a farm producing green crops, vegetables, sugar beet, flax, cereals and cattle-breeding

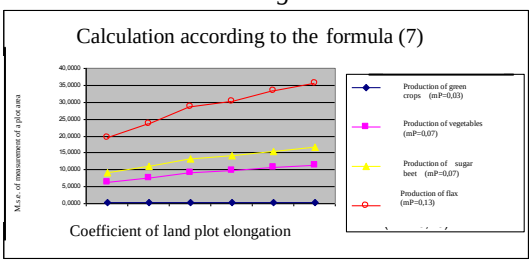

Fig. 3. Diagram of errors of measurement of land plots area made according to the formula (7) for specialization of a farm producing green crops, vegetables, sugar beet, flax, cereals and cattle-breeding

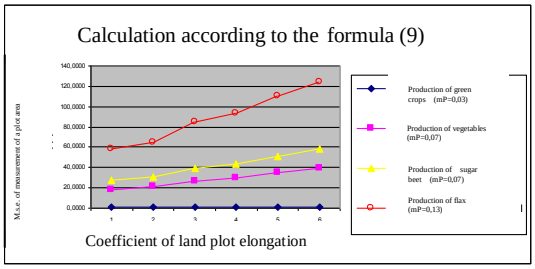

Fig. 4. Diagram of errors of measurement of land plots area made according to the formula (9) for specialization of a farm producing green crops, vegetables, sugar beet, flax, cereals and cattle-breeding

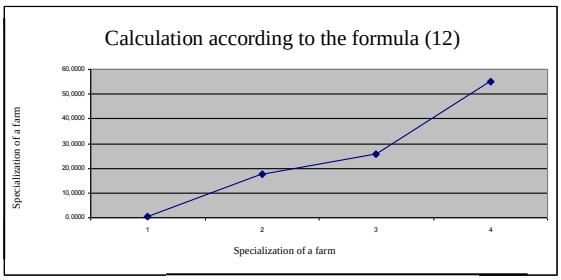

Fig. 5. Diagram of errors of measurement of land plots area made according to the formula (12) for specialization of a farm producing green crops, vegetables, sugar beet, flax, cereals and cattle-breeding

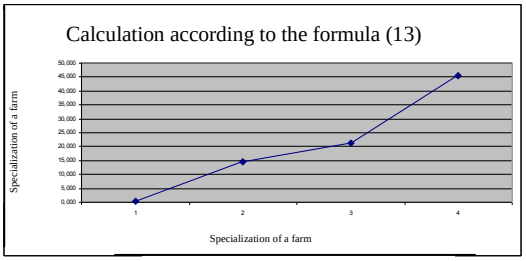

Fig. 6. Diagram of errors of measurement of land plots area according to the formula (13) for specialization of a farm producing green crops, vegetables, sugar beet, flax, cereals and cattle-breeding

These diagrams demonstrate that decrease of errors of measurement of plots area features a method, revealed in the formulas (1) and (9), while for the formulas (5) and (7) the errors increase. Still this fact does not suggest that some formulas are more accurate than other ones.

Results of the calculations prove that declarative requirements as to necessary accuracy of measurement of area does not correspond to calculations made according to the formulas (1), (5), (7) and (9).

Main material

One should mention that the formula (9) is quite complex and can be applied only in case of a posteriori estimation of accuracy, i.e. when all metric parameters of land plot characteristics are determined. However, even in such case methodology, which bases on application of the formula, envisages measurement and adaptation of coordinates.

Let us try to make slight transformation of the expression (9). To do this we use the formula of cotangents and express chorda through sides of a polygon [11]

$$DD_{i-1}^{i+1} = d_{i-1}^2 + d_{i+1}^2 - 2d_{i-1}d_{i+1}\cos\beta, \quad (14)$$

where: d_{i-1} - is a length of a side of a polygon between apexes i-1 and i; d_{i+1} - is a length of a side of a polygon between apexes i+1 and i; β_i – is an angle made by the sides d_{i-1} та d_{i+1}.

Taking into account the expression (14) we have:

$$m_S = \frac{m_i}{2\sqrt{2}}\sqrt{\sum d^2 - \sum d_{i-1}d_{i+1}\cos\beta_i}, \quad (15)$$

where: $\sum d^2$ - is an amount of square of polygon sides.

The formula (15) is more convenient than (9), because its arguments are measured values, but is can be applied only in case of a posteriori estimation of accuracy.

Breaking the strictness of consideration, we suggest that land plots have various configuration, but the angles at their apexes are close to the right ones. As a result:

$$m_S = \frac{m_i}{2\sqrt{2}}\sqrt{\sum d^2}. \quad (16)$$

The formula we have got is rather simple. To estimate accuracy of measurement of land plot area one need only to know the square of its sides.

Analysis of the formula (16) has demonstrated that even if angles at the apexes of a plot make 30°, error of the measurement of area does not exceed a half of its true value.

Reverse transfer is made according to the formula.

$$m_i = \frac{2m_S\sqrt{2}}{\sqrt{\sum d^2}}. \quad (17)$$

Table 8.

M.s.e. of measurement of area of land plots for specialization of a farm according to the formula (16)

m_S (m^2)							Special. of a farm
k	1	2	4	5	7	9	
	0,6708	0,7500	0,9779	1,0817	1,2677	1,4318	1
	18,520	20,706	26,998	29,863	35,000	39,529	2
	27,111	30,311	39,521	43,715	51,235	57,865	3
	58,138	65,000	84,750	93,744	109,87	124,088	4
	489,98	547,72	714,14	789,94	925,82	1045,63	5

As results show, calculations according to the formulas (9) and (16) are identical ones. This fact is confirmed by the derivation of the formula (16)

Diagrams of changes of these dependences is demonstrated (according to the formulas (1) and (7) relatively) at the fig. 4, 5 and 6.

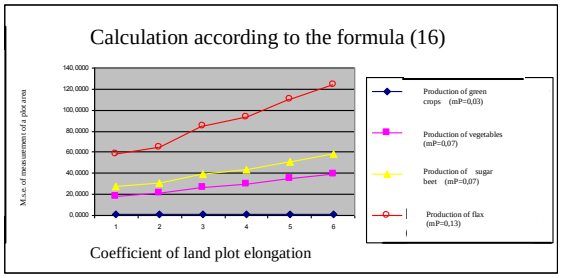

Fig.7. Diagram of errors of measurement of land plots area made according to the formula (16) for specialization of a farm producing green crops, vegetables, sugar beet, flax, cereals and cattle-breeding

Let us consider more complex figure (fig. 8)

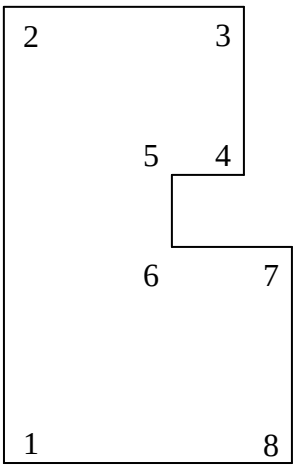

Fig. 8. Shape of a plot

Catalog of coordinates of the plot is presented in the table 7.

Table 7

Coordinates of boundary marks

№	X m	Y m
1	0	0
2	140	0
3	140	30
4	90	30
5	90	20
6	60	20
7	60	40
8	0	40

The real area of the plot makes 4500 m^2.

Let us calculate mean-square errors of the plot area in accordance to the value of mean-square error of areas. Under the condition when mean-square error of boundary marks position makes 0,4 m we get the following results: 41,9 m^2 - according to the formula (1), if k = 4,667, 49 m^2 - according to the formula (5); 13,7 m^2 - according to the formula (7); 40,5 m^2 - according to the formula (10); 24,8 m^2 - according to the formula (12); 35, 9 m^2 - according to the formula (13); and 40,5 m^2 - according to the formula (16).

We have done calculations of real errors of measurement of the plot area under change of coordinates of boundary marks taking into account even distribution of random determined coordinates. 180 variants have been applied. General mean-square error makes 14, 4 m^2. Range of values of real errors is within the frames of -34,601 – 29,060 m^2, which mostly characterizes the formula (7). In such case, the closest values are given to the formulas (10) and (16), and the formulas (12) and (13) do not take into consideration shape of the plot. As it was already mentioned, the formula (16) is handier for a priori estimation of accuracy of measurement of a plot area.

Relative errors of area (S) measurement

In land management it is accepted to estimate errors of measurement of land plot area considering relative errors (relation of mean-square error to absolute value of area), thus we calculate it using the data from table 9.

Table 9

Relative errors of measurement of plot area calculated according to the formula (16)

Sp. of a farm	Coefficient of land plot elongation		
	1	2	4
1	1: 750	1: 670	1: 510
2	1: 3780	1: 3380	1: 2590
3	1: 5530	1: 4950	1: 3800
4	1: 3440	1: 3080	1: 2360
5	1: 3060	1: 2740	1: 2100
Sp. of a farm	Coefficient of land plot elongation		
	5	7	9
1	1: 104350	1: 93330	1: 71580
2	1: 223610	1: 200000	1: 153390
3	1: 223610	1: 200000	1: 153390
4	1: 298140	1: 266670	1: 204520
5	1: 2236070	1: 2000000	1: 1533930

Data of the table demonstrate that relative error increase along with the increase of the coefficient of elongation and decrease of a plot area. Besides, increase happens along with the increase of area. Thus, to estimate accuracy of measurement of area one should apply a differential approach.

Conclusions

To sum up we can make the conclusion that the formulas (16) and (19) are more exact to calculate accuracy of measurement of a plot area than the formula (1). However, the formula (16) applies determined values. To estimate accuracy of measurement of relative errors one should apply a differential approach.

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On the issue of accuracy of area measurement by analytical method

Ye. I. Smirnov

The work presents analysis of accuracy of measurement of land plot area according to different formulas. The author has made a conclusion that for estimation of accuracy of relative errors it is recommended to apply differential approach.