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METHODS TO DETERMINE INTERVALS OF LEVELING RODS WITH ELECTRONIC TACHEOMETER

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Problem. Traditionally, calibration of rods used in the leveling of classes IV and III is performed by the control meter, which must regularly be certified in Kharkiv Meteorological Institute.

We suggest a technique that allows to determine the desired rod intervals for the specified class with the required precision using electronic tacheometer and reflective tape.

The proposed method can also be used for high-precision determination of distances in other applied geodetic problems [4, 7, 8, 9].

Main material.

According to [1] length of meter intervals of checkers rods for Class IV may differ from the nominal value by 1 mm, and decimeter - 0.6 mm. For Class III - 0.5 mm for meters and 0.4 mm - decimeter. So RMS of determination the lengths of these segments should be at least twice smaller than sought values.

Using the formula [5]

$$l = \sqrt{\frac{c \cdot m_{a/b}}{m_\gamma \sqrt{2}} \rho''}, \quad (1)$$

where $m_{a/b}$ – RMS of measuring of length l to rod (at Fig. 1 – a, b), m_γ – RMS of measured angle, c – searched segment on rail (at Fig. 1, e.g. kn), lets find the most optimal distance l from the total station to the rod. For example, for total station Sokkia SET610 $m_{a/b} = 2мм$, $m_\gamma = 5''$, most optimal according to (1) for meter intervals is distance of 7.6 m.

Precision of definable segments c one can find by using [5].

$$m_c(l,\gamma) = \sqrt{2\sin^2\frac{\gamma}{2} \cdot m_{a/b}^2 + l^2\cos^2\frac{\gamma}{2} \cdot \frac{m_\gamma^2}{\rho^2}}. \quad (2)$$

There γ – angle in degrees, ρ = 206265".

For the above-mentioned data for Sokkia SET610 precision of meter intervals of rod from one set of measurements is equal m_{1m} = 0.26 mm, for two-decimeter, for the same distance 7.6 m (according to (1) – 3.4 m) – m_{2dm} 0.19mm.

For Leica TCA2003 $m_{(a/b)}$ = 1mm, m_γ = 1.0". Then, for meter intervals of rod the most optimal, according to (1) is a distance of 12 m .The precision of determination from one set of measurements is equal to m_{1m} = 0.08 mm , and for two-decimeter, for the same distance 12 m (according to (1) – 5.4 m i m_{2dm} = 0.04 mm) – m_{2dm} = 0.19mm. These calculations are performed for the purpose of the definitions of all the intervals from one station.

As you can see from the calculations, the analysis of rod’s interval can be performed by topographic tacheometers such as Sokkia SET610, since the error of the meter intervals even by one set of measurements is almost twice smaller 0.26 mm that 0.5 mm of the desired error of meter intervals.

The study was performed using: precision metrology - Leica TCA2003 and topographical Electronic Theodolite Sokkia SET610, reflective film, rods and four etalon meters. Characteristics of total stations are given in Table 1.

Table 1

Specifications of electronic total stations

Characteristic	Leica TCA2003	Sokkia SET610
RMS of angle measurement	0.5"	5"
RMS of length measurement	1*mm*+ 1*ppm*	2*mm*+ 2*ppm*
Optical zoom	$30^\times$	$30^\times$

Prior to the measurement we identified instrument adjustments for tacheometers of used reflective films. For this we manufactured cylinder with diameter of hole of stands of electronic tacheometers in which the upper part of the cylinder was cut in half-cylinder, so after gluing reflecting surface of the film the axis of the cylinder was not at axis, but was shifted on its thickness. Determination of instrument adjustments performed on tubular marks A, B and C, which are located at a distance of about 5 m among themselves in the alignment (± 5 mm). Measured distance S_{AC}, and then from point B – S_{BA} and S_{BC} on same film glued on this half cylinder. PP = $S_{AC} - (S_{BA} + S_{BC})$.

Determination of instrument adjustments performed for two positions of vertical circle. The instrument adjustment was taken into account in the calculations. So determined PP allows you to measure the distance to the plane of the rod.

Measurement of intervals rod by the control meter and sight during the measurement of distances and angles by electronic tacheometer, was done on strokes at the ends of two-decimeter divisions that was written by well sharpened pencil. During the measurements, rod was installed at the altitude of electronic tacheometer, horizontal $(\pm 5')$. The middle of the rod formed approximately a right angle with the sighting beam. For this distance to the 2nd and 28th decimeter strokes was set identical within 5 mm. Reflective tape for measuring distances was glued at the beginning of 2, 10, 20 and 28 decimeter strokes (Fig.1).

To determine the lengths of the two-decimeter (decimeter) intervals of rod you need to know distances d . These distances (Fig. 1) to the intermediate two-decimeter (decimeter) intervals and accuracy of calculations of distances depending on the precision of operation $m_{a/b}$ and agles – m_γ was calculated as follows. By measured distances a and b and angle γ calculate angle β using formula of tangents:

$$\frac{b+a}{b-a} = \frac{tg\,\frac{1}{2(\beta+\beta_1)}}{tg\,\frac{1}{2(\beta-\beta_1)}}. \qquad (3)$$

As $\beta + \beta_1 = 180^o - \gamma$, so

$$\beta_{(\gamma,a,b)} = 1/2\left(180^o - \gamma + 2arctgn\left(\frac{(b-a)tg\frac{1}{2}\left(180^o-\gamma\right)}{b+a}\right)\right) \qquad (4)$$

RMS m_β of calculation of β could be found by

$$m_\beta = \sqrt{\left(\frac{\partial\beta_{(\gamma,a,b)}}{\partial\gamma}m_\gamma\right)^2 + \left(\frac{\partial\beta_{(\gamma,a,b)}}{\partial a}m_a\right)^2 + \left(\frac{\partial\beta_{(\gamma,a,b)}}{\partial b}mb\right)^2} = $$
$$= \sqrt{\left(\frac{b(a\cos\gamma - b)}{a^2 - 2ab\cos\gamma + \mathrm{b}^2}m_\gamma\right)^2 + \left(\frac{-b\sin\gamma}{a^2 - 2ab\cos\gamma + \mathrm{b}^2}m_a\right)^2 +}$$
$$+ \sqrt{\left(\frac{a\sin\gamma}{a^2 - 2ab\cos\gamma + \mathrm{b}^2}m_b\right)^2}. \qquad (5)$$

Value of d is determined by

$$d_{(a,\beta,\gamma)} = \frac{a\sin\beta}{\sin(\gamma_1 + \beta)}. \qquad (6)$$

RMS of d calculation is

$$m_d = \sqrt{\left(\frac{\partial d_{(a,\beta,\gamma_1)}}{\partial a}m_a\right)^2 + \left(\frac{\partial d_{(a,\beta,\gamma_1)}}{\partial\beta}m_\beta\right)^2 +}$$
$$+ \sqrt{\left(\frac{\partial d_{(a,\beta,\gamma_1)}}{\partial\gamma_1}m_{\gamma_1}\right)^2} = \sqrt{\left(\frac{\sin\beta}{\sin(\beta+\gamma_1)}m_a\right)^2 +} \qquad (7)$$
$$+ \sqrt{\left(\frac{a\sin\gamma_1}{\sin^2(\beta+\gamma_1)}m_\beta\right)^2 + \left(\frac{a\sin\beta}{tg(\beta+\gamma_1)\sin(\beta+\gamma_1)}m_{\gamma_1}\right)^2}.$$

Value of unknown two-decimeter interval e is determined from sine formula

$$e_{(a,\beta,\gamma_1)} = \frac{a\sin\gamma_1}{\sin(\gamma_1+\beta)}. \qquad (8)$$

RMS of two-decimeter (decimeter) interval e is calculated by

$$m_e = \sqrt{\left(\frac{\partial e(\alpha,\beta,\gamma)}{\partial a}m_a\right)^2 + \left(\frac{\partial e(\alpha,\beta,\gamma)}{\partial\beta}m_\beta\right)^2 +}$$
$$+ \sqrt{\left(\frac{\partial e(\alpha,\beta,\gamma)}{\partial\gamma_1}m_{\gamma_1}\right)^2} = \sqrt{\left(\frac{\sin\gamma_1}{\sin(\beta+\gamma_1)}m_a\right)^2 +} \qquad (9)$$
$$+ \sqrt{\left(\frac{\alpha\sin\gamma_1}{tg(\beta+\gamma_1)\cdot\sin(\beta+\gamma_1)}m_\beta\right)^2 +}$$
$$+ \sqrt{\left(\frac{\alpha\sin\beta}{\sin^2(\beta+\gamma_1)}m_{\gamma_1}\right)^2}.$$

Performing calculations in accordance with the above formula we can see that the accuracy of calculation is almost equal to the precision of measurement of meter intervals and shall not affect the accuracy of the calculations of two-decimeter (decimeter) intervals. For example, if intervals a and b are measured with accuracy of 2 mm, and angle γ is measured with accuracy of 5", then length d for two-decimeter interval would be obtained with accuracy m_d = 2.07mm.

This error in determining the distances leads to an error in calculating two-decimeter interval of 0.006 mm, which has virtually no effect on the accuracy of calculation of intervals.

Measurements of lengths and angles with tacheometer were done simultaneously. There were 4 measurement sets done by tacheometer Leica and 5 by Sokkia. Tacheometers were located at a distance of about 10 m from the rod. First measurements on all strokes were performed with KL clockwise, and then with KP – counter-clockwise. This was one set of measurements. On each stroke two sightings for each position of the circle were performed. The difference between the two sightings does not exceed 2". Similarly, distances were measure with two circles.

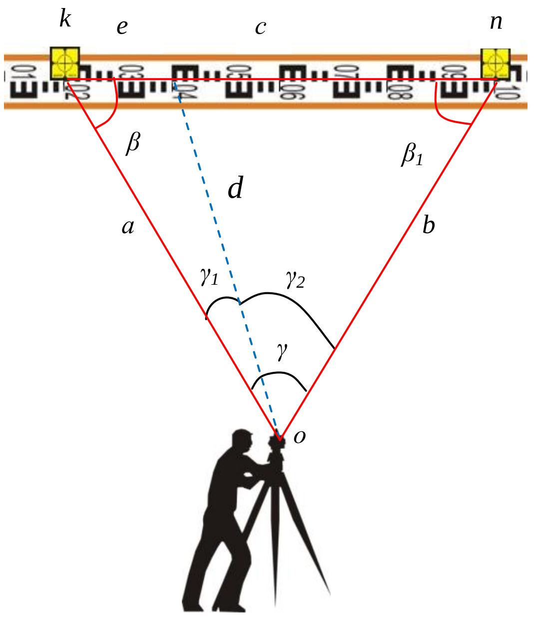

Figure 1. Scheme of rod interval calculation.

To check the precision of the proposed method, definable intervals on rod were measured with four control meters in forward and reverse directions. RMS of measured intervals calculated by the Bessel formula did not exceed 0.06 mm for two-decimeter intervals and 0.08 mm - for meter intervals.

The results of the comparisons of meter intervals measured with control lines and the proposed method for total station Leica TCA2003 are presented in Table 2.

Table 2

Comparison of meter rod intervals measured with 4 control meters with different number of sets of measurements with Leica TCA2003

Rod interval, mm	Average interval measured with control meters, mm	Difference (example – control meters) set 1 (PR1), mm	Difference (example – control meters) (PR1+PR2)/2, mm	Difference (example – control meters) (PR1+PR2+PR3)/3, mm	Difference (example – control meters) (PR1+PR2+PR3+PR4)/4, mm
200					
1000	800.18	+0.09	+0.04	-0.01	-0.02
2000	999.73	+0.06	-0.02	-0.04	-0.03
2800	799.68	-0.09	+0.02	+0.01	+0.01

Table 3

Comparison of meter rod intervals measured with 4 control meters with different number of sets of measurements with Sokkia SET610

Rod interval, mm	Average interval measured with control meters, mm	Difference (example – control meters) set 1 (PR1), mm	Difference (example – control meters) (PR1+PR2)/2, mm	Difference (example – control meters) (PR1+PR2+PR3)/3, mm	Difference (example – control meters) (PR1+PR2+PR3+PR4)/4, mm	Difference (example – control meters) (PR1+PR2+PR3+PR4+PR5)/5, mm
200						
1000	800.18	+0.14	+0.12	+0.05	+0.10	-0.02
2000	999.73	-0.05	-0.05	-0.06	-0.04	+0.02
2800	799.68	+0.15	+0.13	+0.13	+0.07	+0.05

Table 4

Comparison of two-decimeter rod intervals measured with 4 control meters with different number of sets of measurements with Leica TCA2003

Rod interval, mm	Average interval measured with control meters, mm	Difference (example – control meters) set 1 (PR1), mm	Difference (example – control meters) (PR1+PR2)/2, mm	Difference (example – control meters) (PR1+PR2+PR3)/3, mm	Difference (example – control meters) (PR1+PR2+PR3+PR4) /4, mm
200					
400	199.98	-0.10	-0.03	-0.05	-0.07
600	200.13	-0.02	+0.01	+0.03	+0.06
800	200.02	+0.16	+0.10	+0.05	+0.01
1000	200.05	+0.05	-0.04	-0.03	-0.03
1200	199.92	+0.06	+0.01	+0.03	+0.05
1400	200.02	-0.11	-0.04	-0.05	-0.04
1600	199.88	-0.08	+0.00	+0.00	+0.00
1800	200.02	+0.01	-0.01	+0.00	+0.00
2000	199.88	+0.19	+0.01	-0.02	-0.03
2200	199.94	-0.06	+0.00	-0.01	-0.01
2400	199.90	-0.10	-0.03	-0.01	-0.02
2600	199.95	+0.03	+0.02	+0.02	+0.02
2800	199.89	+0.04	+0.03	+0.01	+0.01

Table5

Comparison of two-decimeter rod intervals measured with 4 control meters with different number of sets of measurements with Sokkia SET610

Rail interval, mm	Average interval measured with control meters, mm	Difference (example – control meters) set 1 (PR1), mm	Difference (example – control meters) (PR1+PR2)/2, mm	Difference (example – control meters) (PR1+PR2+PR3)/3, mm	Difference (example – control meters) (PR1+PR2+PR3 +PR4)/4, mm	Difference (example – control meters) (PR1+PR2+PR3+ PR4+PR5)/5, mm
200						
400	199,98	-0.08	-0.08	-0.12	-0.02	-0.06
600	200,13	+0.16	+0.17	+0.13	+0.09	+0.06
800	200,02	-0.01	-0.04	-0.02	-0.01	-0.03
1000	200,05	+0.06	+0.07	+0.04	+0.04	+0.02
1200	199,92	+0.16	+0.13	+0.13	+0.08	+0.10
1400	200,02	-0.03	-0.04	-0.01	+0.00	+0.02
1600	199,88	-0.08	-0.10	-0.11	-0.04	-0.05
1800	200,02	+0.03	+0.05	+0.01	+0.03	+0.03
2000	199,88	-0.14	-0.09	-0.11	-0.11	-0.10
2200	199,94	+0.06	+0.09	+0.12	+0.10	+0.06
2400	199,90	+0.07	+0.00	+0.01	-0.02	+0.01
2600	199,95	+0.01	+0.03	+0.02	+0.06	+0.04
2800	199,89	+0.10	+0.06	+0.07	+0.00	+0.00

Conclusions.

As we see, the results obtained in Tables 2-5 are somewhat different from pre-calculated. This is both due to the precision of the intervals measured with control lines, and because of the fact that the actual distance to the rod for both tacheometers, for meter and two-decimeter intervals, was equal to 10 m, and not calculated by the formula (1).

To get definable intervals precision, three times greater than required by the instruction, for example, 0.4 mm for decimeter rod intervals, even for a 10 m distance, simply follow the two sets of measurements with metrological tacheometer Leica TCA2003 and three sets – with topographic Sokkia SET610.

For more Leica TCA2003 sets of measurements, and distances to the rod calculated with formula (1), these studies can be performed for the rods used in the first and second classes of leveling.

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Methods to determine intervals of leveling rods with electronic tacheometer.

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The article examines the method of comparation of leveling rods that are used in the leveling of classes III and IV, using an electronic tacheometer and reflective tapes. For optimally calculated with our proposed formulas, distance to rail, depending on the size of the segment defined by rail and accuracy of electronic tacheometer, you can get corresponding accuracy of these rails comparation. Regardless of the desired interval values, the distance, in the proposed method we measured to only a few rail intervals with an electronic tacheometer. For the remaining intervals distances were calculated with our proposed formulas.